Kinetic Approach to Temperature Relaxation in Dense Plasmas

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Abstract. The relaxation of nonideal two-temperature plasmas is investigated with a kinetic approach. First the energy transfer between the electrons and ions is described using different approximations: the energy transfer through classical collisions (Landau-Spitzer approach) is reviewed; quantum diffraction and strong collisions are included by applying the quantum Boltzmann equation; the influence of collective modes is considered on the basis of the Lenard-Balescu equation (coupled modes) and with the Fermi-Golden-Rule approach (independent electron and ion modes). Finally, the evolution of the species temperature is investigated. In nonideal plasmas, changes in the correlation energy have to be taken into account during the relaxation. It is demonstrated that ionic correlations can significantly influence the relaxation (particularly the evolution of the ion temperature).

1. Introduction

Most experimental techniques that are designed to create dense plasmas produce systems in extreme nonequilibrium states since the energy is mostly coupled into one species. Lasers and particle beams transfer for instance almost the entire energy to the electrons while shock waves produce plasmas with energetic heavy particles and cold electrons. Subsequently, the properties of the plasma are governed by the relaxation towards the equilibrium state. The relaxation times are also of importance for measurements of equilibrium properties since they determine the minimum time delay between creation and probing.

In dense plasmas, most properties have reached equilibrium values after a few hundred femtoseconds. In particular, the particles have established Fermi/Maxwell distributions[1], but the species temperatures are still different. During the following stage the relaxation is determined by the energy transfer between electrons and ions. However, the temperature equilibration can also drive changes in the plasma composition or the correlation energy. In turn, the thermal energy is then modified by the time-dependent binding [3, 4, 5] or correlation energies [6].

The energy transfer between electrons and ions was first described in the seminal works of Landau and Spitzer (LS approach) [7, 8], where hot, weakly coupled plasmas were considered. Due to the assumption of classical Coulomb collisions, *ad hoc* cutoffs had to be introduced. Comparisons with numerical simulations showed slightly higher or lower energy transfers depending on the applied cutoffs [9, 10]. To include strong electron-ion collisions and to also

[1] Although the momentum distribution is modified in correlated quantum systems, the deviations from the ideal gas form are very small [1, 2] and will therefore be neglected during the subsequent temperature equilibration.

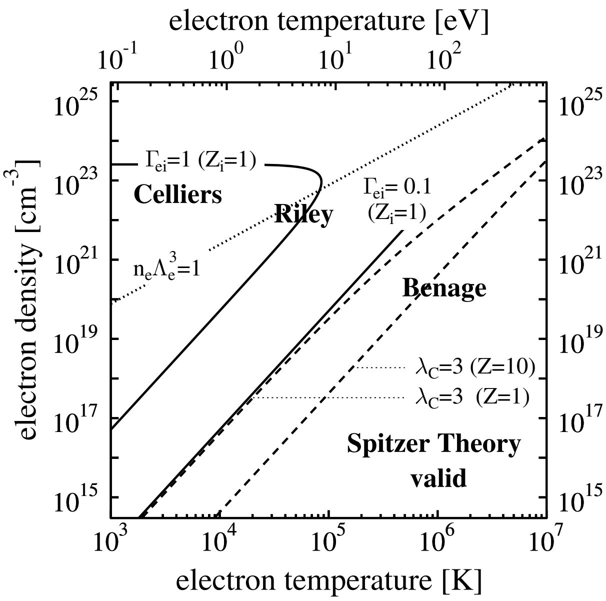

Figure 1. Parameter region of interest in the density-temperature plane. The names indicate recent dense plasma experiments [14, 15, 16, 17]. The lines show parameters with constant electron-ion coupling parameter Γ_{ei}, electron degeneracy $n_e\Lambda_e^3$, and Coulomb logarithm λ_C. Γ_{ei} contains also degeneracy effects which create a bending of the curve at high densities; in this region, the horizontal line of $\Gamma_{ei}=1$ coincides approximately with $r_s=0.74$.

avoid the ambiguity in the LS approach, a quantum kinetic approach based on a Boltzmann-type collision integral was applied [11, 3, 12]. Due to the correct description of binary collisions, the break-down of the LS theory in strongly coupled plasmas could here also be avoided. Nevertheless, this Boltzmann approach still neglects the collective behavior of the plasma, i.e., dynamic screening and collective excitations (plasmons). Recently, Dharma-wardana and Perrot developed energy transfer models that consider such collective modes [13]. It turned out that the consideration of coupled collective modes strongly reduces the energy transfer between electrons and ions.

Compared to the LS approach, recent experimental results showed also strong indications of a much smaller energy transfer in dense plasmas. Such results have been found in laser- [14] as well as shock-produced [15, 16] plasmas, i.e. for $T_e \gg T_i$ and $T_e \ll T_i$. The plasma condition in these experiments are indicated in Fig. 1 by the names of the first authors[2]. Obviously, these plasmas are strongly coupled and partially/highly degenerate. Therefore, the LS approach, which is only valid for Coulomb logarithms larger than three, has to be generalized towards strong electron-ion coupling and degenerate systems.

The quantum kinetic theory provides well-developed approximation schemes that allow for the consideration of strong coupling, collective excitation, and degeneracy. These schemes are here applied to describe the energy transfer in two-temperature plasmas. In section 3, the energy transfer due to binary collisions is discussed. First the LS approach is reviewed. Then expressions for the energy transfer rates valid for strong electron-ion scattering are derived on the basis of the quantum Boltzmann equation. Collective modes in weakly coupled plasmas can be described with the Lenard-Balescu equation. In section 4, this kinetic equation is used to describe the energy transfer through coupled modes. Furthermore, the Fermi-Golden-Rule approach is discussed. Finally, the influence of the time-dependent correlation energies on the temperature equilibration is investigated in section 5.

2. Basic Kinetic Description of Temperature Equilibration in Nonideal Plasmas

In the following, the relaxation of a two-temperature electron-ion system is investigated: electrons and ions have established equilibrium distributions that allow to define electron and

[2] More measurements have been done using excited semiconductors with hot electrons. However, the conduction electrons equilibrate here with the lattice phonons, not with the temperature of an gas- or fluid-like ion component.

ion temperatures T_e and T_i, respectively. The ions are considered to have a fixed charge Z_i, i.e., ionization and recombination events are not included (for the influence of such reactions, see Refs. [3, 4, 5]). The build up of correlations, in particular the ion-ion correlations, is also considered to be completed, i.e., the ion-ion correlation energy is in equilibrium with the ion temperature T_i.

For weakly coupled plasmas, the internal energy of the system is approximately given by the sum of the electron and ion kinetic energies. Accordingly, the relaxation is fully described by the energy transfer between the subsystems. In the case of nondegenerate plasmas, one has to solve the system of equations

$$\frac{3}{2} n_e k_B \frac{\partial}{\partial t} T_e = -\frac{\partial}{\partial t} E_{e\to i}^{trans} \qquad \text{and} \qquad \frac{3}{2} n_i k_B \frac{\partial}{\partial t} T_i = \frac{\partial}{\partial t} E_{e\to i}^{trans} \,. \tag{1}$$

The energy transfer rates can be derived from kinetic equations. Starting with the definition of the average kinetic energy, a balance equation for the energy of the particles of species a can be obtained

$$\frac{\partial}{\partial t} E_a = \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3} \frac{p^2}{2m_a} \frac{\partial}{\partial t} f_a(\boldsymbol{p}, t) \tag{2}$$

$$= \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3} \frac{p^2}{2m_a} I(p, t) \,. \tag{3}$$

In the second step, the kinetic equation with a collision integral $I(p, t)$ was used (homogeneous plasmas are considered).

These considerations become more complicated for dense, nonideal systems. Here, the internal energy also includes correlation contributions

$$E^{tot} = E_e^{kin} + U_{ee}^{ex} + E_i^{kin} + U_{ii}^{ex} + U_{ei}^{ex} \,, \tag{4}$$

where U_{ee}^{ex}, U_{ii}^{ex}, and U_{ei}^{ex} denote the electron-electron, ion-ion, and electron-ion excess (or correlation) energies, respectively. In strongly coupled plasmas, these correlation contributions can be as important or even larger than the kinetic energy terms. Due the occurrence of the electron-ion term, it is furthermore not obvious how to define the "electron" and "ion" subsystems. Therefore, four questions have to be answered before a description of temperature relaxation in strongly coupled plasmas is obtained:

- How can the energy of the electrons and the ionic energy be defined?
- What is the (quasi)-equation of state for two-temperature plasmas?
- How get changes in one term of the internal energy (4), e.g. a decrease of electron temperature, redistributed between the other four terms?
- How do correlations influence the energy transfer between the electron and ion subsystems?

The energy transfer rates are investigated in the next two sections. Then a model for the important case of temperature equilibration in plasmas with hot electrons and strongly coupled ions is introduced in the last section.

3. Energy Transfer Due to Binary Collisions

3.1. Landau-Spitzer Approach

Let us briefly review the Landau-Spitzer approach to temperature relaxation. In their works [7, 8], they consider weakly coupled electrons and ions that transfer energy through small angle Coulomb collisions. The electron trajectories are described by straight lines and characterized by the impact parameter b and the impact energy. By summing over all impact parameters, the energy or momentum transfer cross section is obtained in the form [18]

$$Q_{ei}^T(v) = \frac{4\pi Z_i e^4}{m_e v^4}\lambda_C \,, \tag{5}$$

where $\boldsymbol{v} = \boldsymbol{v}_e - \boldsymbol{v}_i$ is the velocity of relative motion between the scattering electron and ion. λ_C is the well-known Coulomb logarithm, i.e., the logarithm of the ratio of the maximum and minimum considered impact parameters

$$\lambda_C = \ln\left(\frac{b_{max}}{b_{min}}\right) = \ln\left(\frac{\lambda_D}{\sqrt{\rho_\perp^2 + \lambda_{dB}^2}}\right) . \tag{6}$$

In the second step, the “usual” cutoffs are used: the largest impact parameter is estimated by the electron Debye length $\lambda_D = (k_B T_e/4\pi e^2 n_e)^{1/2}$. To model quantum diffraction effects, a quadratic interpolation between the deBroglie wave length $\lambda_{dB} = \hbar/m_e v_{th}$ and the distance of closest approach $\rho_\perp = Z_i e^2/m_e v_{th}^2$ is used for the lower cutoff. $v_{th} = (k_B T/m_e)^{1/2}$ denotes here the thermal velocity of the electrons. It should be mentioned again that these cutoffs are physically motivated, but by no means derived!

Obviously, the description above fails for $\lambda_C < 0$ which occurs in dense, strongly coupled plasmas. This break-down can be traced back to the concept of straight line trajectories. Indeed, if one uses the known hyperbolic trajectories for the Coulomb scattering, the integral for the momentum transfer becomes convergent at the lower boundary and, therefore, the lower cutoff can be set to be zero. The corresponding Coulomb logarithm has the form

$$\lambda_C = \frac{1}{2}\ln\left(1 + \frac{b_{max}^2}{b_{ref}^2}\right) . \tag{7}$$

Here, $b_{ref} = \rho_\perp$ denotes a reference impact parameter which can be also used in the form $b_{ref} = (\rho_\perp^2 + \lambda_{dB}^2)^{1/2}$ to model quantum effects.

With these results for the energy transfer cross section, the total energy transfer between the electron and ion subsystems follows by averaging with the Maxwellian velocity distributions. Finally, the temperature evolution can be expressed by

$$\frac{\partial}{\partial t}T_e = \sum_\alpha \frac{T_\alpha - T_e}{\tau^{e\alpha}} \,, \tag{8}$$

where the sum runs over all ion species. For the relaxation time τ^{ei}, one obtains the result [7, 8]

$$\tau^{e\alpha} = \frac{3 m_\alpha m_e}{8\sqrt{2\pi} n_\alpha Z_\alpha^2 e^4 \lambda_C}\left(\frac{k_B T_e}{m_e} + \frac{k_B T_\alpha}{m_\alpha}\right)^{3/2} . \tag{9}$$

3.2. Quantum Boltzmann Approach

Now a quantum kinetic approach, that also accounts for strong electron-ion scattering, will be applied to derive the energy transfer rates in the binary collision approximation. Due to the full quantum mechanical treatment of electron-ion collisions and the consideration of statically screened interactions, any arbitrary cutoffs can be avoided. The basis of this approach is given by the electron-ion collision integral of the quantum Boltzmann equation for nondegenerate plasmas [19, 20]

$$\begin{aligned} I_{ei}(p,t) = \frac{1}{V\hbar}\sum_i \int \frac{d\boldsymbol{p}'}{(2\pi\hbar)^3}\frac{d\overline{\boldsymbol{p}}}{(2\pi\hbar)^3}\frac{d\overline{\boldsymbol{p}}'}{(2\pi\hbar)^3}\; 2\pi\delta\Big(E_e(\boldsymbol{p}) + E_i(\boldsymbol{p}') - E_e(\overline{\boldsymbol{p}}) - E_i(\overline{\boldsymbol{p}}')\Big) \\ \times \left|\langle \boldsymbol{p}\,\boldsymbol{p}' \,|\, \mathrm{T}_{ei}^R \,|\, \overline{\boldsymbol{p}}'\,\overline{\boldsymbol{p}}\rangle\right|^2 \Big\{f_e(\overline{\boldsymbol{p}},t) f_i(\overline{\boldsymbol{p}}',t) - f_e(\boldsymbol{p},t) f_i(\boldsymbol{p}',t)\Big\} . \end{aligned} \tag{10}$$

Here, the sum runs again over all ion species. $E_a(p) = p^2/2m_a$ denotes the kinetic energy and f_a is the momentum distribution of species a. The scattering probability is described by the matrix elements of the retarded T-matrix T^R_{ei}.

The energy transfer rate is then obtained by using this collision integral in the balance equation (3). To derive an explicit expression, a variable transformation to the momenta of center of mass and relative motion

$$\boldsymbol{P} = \boldsymbol{p} + \boldsymbol{p}' \qquad \text{and} \qquad \boldsymbol{q} = \frac{\mu_{ei}}{m_e}\boldsymbol{p} - \frac{\mu_{ei}}{m_i}\boldsymbol{p}' \tag{11}$$

is done. Using the energy conservation and the momentum conservation in the collision, which is included in the T-matrix, four integrations can be easily performed. With the new variables, the matrix elements of the T-matrix determine the differential scattering cross section

$$\frac{d\sigma_{ei}(q,\theta)}{d\Omega} = \frac{(2\pi)^4\hbar^2\mu_{ei}^2}{(2\pi\hbar)^6}\Big|\langle \boldsymbol{q}\,|\,\mathrm{T}^R_{ei}\,|\,\overline{\boldsymbol{q}}\rangle\Big|^2_{q=\overline{q}} \ . \tag{12}$$

Furthermore, it is useful to introduce the transport cross section by

$$Q^T(q) = 2\pi\int_0^\infty d\theta\,\sin\theta(1-\cos\theta)\frac{d\sigma_{ei}(q,\theta)}{d\Omega} \ . \tag{13}$$

With this definition and the known Boltzmann-like distribution functions for both species, one can analytically perform all integration except one and finally obtains for the energy transfer rate

$$\begin{aligned}\frac{\partial}{\partial t}E^{trans}_{e\to i} = &-\frac{16\,\mu_{ei}\,n_i\,n_e}{\sqrt{2\pi}\,m_i}\frac{k_B(T_e-T_i)}{(2m_e k_B T_e)^3}\left(\frac{k_B T_e}{m_e}+\frac{k_B T_i}{m_i}\right)^{1/2}\\ &\times\int_0^\infty dq\,q^5\,Q^T_{ei}(q)\,\exp\left(-\frac{q^2}{2m_e k_B T_e}\right) \ ,\end{aligned} \tag{14}$$

The main input quantity in this expression is the transport cross section. For an exact calculation, a phase shift analysis is applied. Here, the cross section is computed from momentum and angular momentum dependent scattering phase shifts $\delta_l(k)$ [21]

$$Q^T(q) = \frac{4\pi}{q^2}\sum_{l=0}^{\infty}(l+1)\,\sin^2\left(\delta_l(q)-\delta_{l+1}(q)\right) \ . \tag{15}$$

The scattering phase shifts are obtained from numerical solutions of the Schrödinger equation where statically screened Coulomb potential

$$V(r) = -\frac{Z_i e^2}{r}\exp(-r/\lambda_D) \tag{16}$$

is used to describe the electron-ion interaction. For weakly coupled plasmas or large relative momenta k, the transport cross section can be also used in first Born approximation [21].

3.3. Results for the Energy Transfer Rates

As already mentioned, the quantum kinetic scheme for the energy transfer rates (14) with the cross section (15) avoids all the usual approximations. The two assumptions are that electrons and ions exchange energy only through binary collisions and that the electron-ion interaction is given by the statically screened Coulomb potential (16). The results of this approach can be

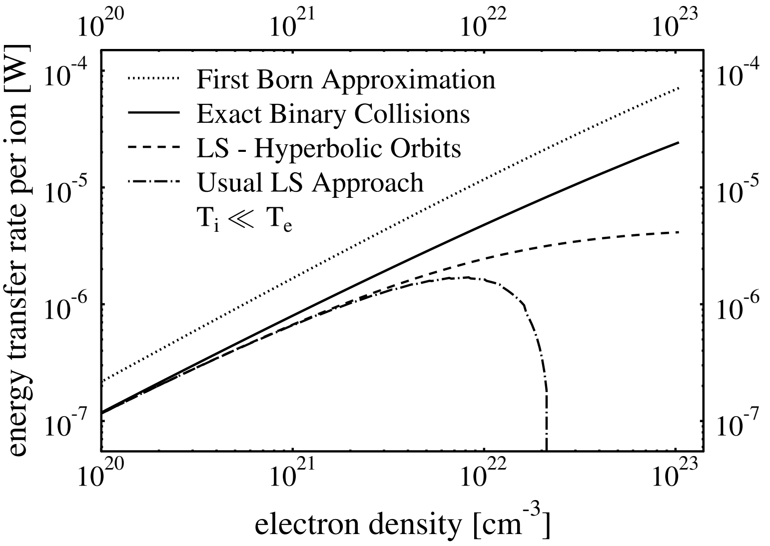

Figure 2. Energy transfer rates in different approximations versus plasma density. The system is an aluminum plasma with an electron temperature of $T_e = 5 \times 10^5\,\mathrm{K}$. The ion charge is fixed to be $Z_i = 10$.

therefore used as a reference for a discussion of the validity of further approximations. It can be particularly tested under which conditions the numerical effort can be significantly reduced since the easy-to-use LS scheme is applicable.

Results for the energy transfer rates between the electron and ion subsystems in different approximations are shown in Fig. 2. The Born approximation for the transport cross section leads, for the considered dense plasmas, to an overestimation of the energy transfer rate (14). Agreement with the full binary collision theory, that uses cross sections calculated by the phase shift approach (15), is only reached for much higher electron temperatures. The usual LS approach shows the known break-down at dense, more strongly coupled plasmas. Here, the Coulomb logarithm (6) becomes negative and the approach is no more applicable. The LS-like scheme that uses hyperbolic electron trajectories (7) avoids this behavior, but still underestimates the energy transfer. Nevertheless, both LS approaches give good results in the weakly coupled, low density regime.

4. Consideration of Collective Modes

The major short-coming of the theory presented in the previous section is that each electron and ion is considered as an independent particle which undergoes only binary collisions. The collective behavior of the plasma particles, in particular plasma waves, are therefore not included. How such collective modes can be included in the theory and how they influence the energy transfer between electrons and ions is investigated in this section on two approximation levels: first the fully coupled electron-ion system is considered. Since the numerical evaluation of the derived expression for the energy transfer rates is very complicated, the Fermi-Golden-Rule (FGR) as the easiest approach that includes collective electron and ion modes is then considered in the next subsections.

4.1. Coupled Collective Modes

There exists a well-developed approach that describes the collective behavior of weakly coupled plasmas. The corresponding kinetic equation is the Lenard-Balescu equation [22, 23], where the dynamic response of the particles is described in random phase approximation (RPA). Since electrons and ions are treated as one system in this approach, the influence of the electrons on the ion response and vice versa (coupled collective modes) is naturally included. The basis for the derivation of the energy transfer rates on this level will be here the quantum version of the

Lenard-Balescu equation with the collision integral given by [19]

$$\begin{aligned} I_{ei}(p,t) \;=\; & \frac{1}{\hbar}\sum_i \int \frac{d\boldsymbol{p}'}{(2\pi\hbar)^3}\frac{d\overline{\boldsymbol{p}}}{(2\pi\hbar)^3}\frac{d\overline{\boldsymbol{p}}'}{(2\pi\hbar)^3} \left|\frac{\mathrm{V}_{ei}(\boldsymbol{p}-\overline{\boldsymbol{p}})}{\varepsilon^R\left(\boldsymbol{p}-\overline{\boldsymbol{p}},E_e(p)-E_e(\overline{p}),t\right)}\right|^2 \\ \times\; & 2\pi\delta\Big(E_e(\boldsymbol{p})+E_i(\boldsymbol{p}')-E_e(\overline{\boldsymbol{p}})-E_i(\overline{\boldsymbol{p}}')\Big)\,(2\pi\hbar)^3\,\delta\Big(\boldsymbol{p}+\boldsymbol{p}'-\overline{\boldsymbol{p}}-\overline{\boldsymbol{p}}'\Big) \\ \times\; & \Big\{f_e(\overline{\boldsymbol{p}},t)f_i(\overline{\boldsymbol{p}}',t)\,[1-f_e(\boldsymbol{p},t)]\,[1-f_i(\boldsymbol{p}',t)] \\ & \qquad -f_e(\boldsymbol{p},t)f_i(\boldsymbol{p}',t)\,[1-f_e(\overline{\boldsymbol{p}},t)]\,[1-f_i(\overline{\boldsymbol{p}}',t)]\Big\}\,. \end{aligned} \tag{17}$$

Here, the same notation is used as for the Boltzmann approach (10). Furthermore, $V_{ab}(k) = 4\pi Z_a Z_b e^2/k^2$ denotes the pure Coulomb potential between particle of species a and b and the retarded dielectric function is given in RPA by

$$\varepsilon^R(\boldsymbol{p},E,t) = 1+\sum_a V_{aa}(p)\,\chi^0_{aa}(\boldsymbol{p},E,t)\,. \tag{18}$$

The dielectric function in RPA was here expressed in terms of the density response function of free particles [24]

$$\chi^0_{aa}(\boldsymbol{p},E,t) = \int \frac{d\boldsymbol{p}'}{(2\pi\hbar)^3}\frac{f_a(\boldsymbol{p}'+\boldsymbol{p},t)-f_a(\boldsymbol{p}',t)}{E+E_a(\boldsymbol{p}')-E_a(\boldsymbol{p}'+\boldsymbol{p})+i\epsilon}\,, \tag{19}$$

which is determined by the time-dependent distribution function.

The energy transfer rates are now obtained by inserting the collision integral (17) into the relation (3). For practical reasons, it is now useful to introduce the momentum and the energy transfer during the collision, i.e.,

$$\boldsymbol{k} = \boldsymbol{p}-\overline{\boldsymbol{p}} \qquad \text{and} \qquad \omega = E_e(p)-E_e(\overline{p})\,, \tag{20}$$

and use the relations between Fermi-distributions of the plasma species and the Bose functions $n_B^a(\omega) = [\exp(\hbar\omega/k_BT_a)-1]^{-1}$

$$f_a(\boldsymbol{p})\,[1-f_a(\boldsymbol{p}+\boldsymbol{k})] = [f_a(\boldsymbol{p}+\boldsymbol{k})-f_a(\boldsymbol{p})]\;n_B^a\Big(E_a(\boldsymbol{p})-E_a(\boldsymbol{p}+\boldsymbol{k})\Big)\,, \tag{21}$$

$$f_a(\boldsymbol{p}+\boldsymbol{k})\,[1-f_a(\boldsymbol{p})] = [f_a(\boldsymbol{p})-f_a(\boldsymbol{p}+\boldsymbol{k})]\;n_B^a\Big(E_a(\boldsymbol{p}+\boldsymbol{k})-E_a(\boldsymbol{p})\Big)\,. \tag{22}$$

The set of distribution functions in the collision integral (17) becomes then

$$\begin{aligned} \Big\{f's\Big\} = & \;[f_e(\overline{\boldsymbol{p}}+\boldsymbol{k})-f_e(\overline{\boldsymbol{p}})]\,[f_i(\boldsymbol{p}')-f_i(\boldsymbol{p}'+\boldsymbol{k})] \\ & \qquad \Big[n_B^e(-\omega)\,n_B^i(\omega)-n_B^e(\omega)\,n_B^i(-\omega)\Big]\,, \end{aligned} \tag{23}$$

and we obtain for the energy transfer rate

$$\begin{aligned} \frac{\partial}{\partial t}E^{trans}_{e\to i} \;=\; & -\frac{1}{\hbar}\sum_i\int\frac{d\overline{\boldsymbol{p}}}{(2\pi\hbar)^3}\frac{d\boldsymbol{k}}{(2\pi\hbar)^3}\frac{d\omega}{2\pi}\;E_e(\overline{\boldsymbol{p}}+\boldsymbol{k})\;\left|\frac{\mathrm{V}_{ei}(\boldsymbol{k})}{\varepsilon^R(\boldsymbol{k}\omega,t)}\right|^2\;\Big[f_e(\overline{\boldsymbol{p}}+\boldsymbol{k})-f_e(\overline{\boldsymbol{p}})\Big] \\ \times\; & 2\pi\delta\Big(\omega-E_e(\overline{\boldsymbol{p}}+\boldsymbol{k})+E_e(\overline{\boldsymbol{p}})\Big)\Big[n_B^e(-\omega)\,n_B^i(\omega)-n_B^e(\omega)\,n_B^i(-\omega)\Big] \\ \times\; & \int\frac{d\boldsymbol{p}'}{(2\pi\hbar)^3}\,[f_i(\boldsymbol{p}')-f_i(\boldsymbol{p}'+\boldsymbol{k})]\;2\pi\delta\Big(\omega-E_i(\boldsymbol{p}'+\boldsymbol{k})+E_i(\overline{\boldsymbol{p}}')\Big)\,. \end{aligned} \tag{24}$$

The last line in this expression is just the definition of the imaginary part of the free density response function for the ionic subsystem $\operatorname{Im}\chi^0_{ii}(k\omega)$. The difference of Bose functions in the second line determines now the direction of the energy transfer. After a change of variables defined by

$$\omega' = -\omega, \qquad \boldsymbol{k}' = -\boldsymbol{k}, \qquad \text{and} \qquad \boldsymbol{p}' = \overline{\boldsymbol{p}} - \boldsymbol{k}' \tag{25}$$

in the second term, this term has the same form as the first one, except that the energy in front of the screened potential is $E_e(\overline{\boldsymbol{p}})$ instead of $E_e(\overline{\boldsymbol{p}} - \boldsymbol{k})$. Using the energy conserving δ-function, the difference of energies is

$$E_e(\overline{\boldsymbol{p}} + \boldsymbol{k}) - E_e(\overline{\boldsymbol{p}}) = \omega\,. \tag{26}$$

Now the remaining electron distributions and the energy conserving δ-function also give the definition of an imaginary part of the free density response function, i.e. $\operatorname{Im}\chi^0_{ee}(k\omega)$. The energy transfer rate is now given by

$$\begin{aligned}\frac{\partial}{\partial t}E^{trans}_{e\to i} = &-4\hbar\sum_i\int\frac{d\boldsymbol{k}}{(2\pi\hbar)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\;\omega\left|\frac{\mathrm{V}_{ei}(k)}{\varepsilon^R(k\omega,t)}\right|^2\\ &\times\operatorname{Im}\chi^0_{ee}(k\omega)\,\operatorname{Im}\chi^0_{ii}(k\omega)\,n^e_B(-\omega)\,n^i_B(\omega)\,.\end{aligned} \tag{27}$$

If we now use the symmetry of the Bose and density response functions ($[1/2 - n_B(\omega)]$ and $\operatorname{Im}\chi^0_{aa}(\omega)$ are odd functions), the energy transfer between the electron and ion subsystems can be also written in the form

$$\begin{aligned}\frac{\partial}{\partial t}E^{trans}_{e\to i} = &-4\hbar\sum_i\int\frac{d\boldsymbol{k}}{(2\pi\hbar)^3}\int_{0}^{\infty}\frac{d\omega}{2\pi}\;\omega\left|\frac{\mathrm{V}_{ei}(k)}{\varepsilon^R(k\omega,t)}\right|^2\\ &\times\operatorname{Im}\chi^0_{ee}(k\omega)\,\operatorname{Im}\chi^0_{ii}(k\omega)\left[n^e_B(\omega) - n^i_B(\omega)\right].\end{aligned} \tag{28}$$

This expression gives the energy transfer including the effects of coupled collective modes in the electron-ion system. It is applicable for weakly coupled plasmas without any restriction with respect to the degeneracy of the plasma.

The expression (28) is equivalent to the coupled mode formula given by Dharma-wardana & Perrot [13] even though it was derived here for weakly coupled plasmas. The extensions toward strong coupling in the formula given in Ref. [13] are the use of a pseudo-potential for the electron-ion interactions to model strong scattering effects and a generalized dielectric function that includes local field corrections.

4.2. Fermi-Golden-Rule

Since an evaluation of the coupled mode expression (28) is extremely difficult, the Fermi-Golden-Rule (FGR) approach to temperature equilibration will be considered here, too. The FGR expression to the energy transfer rates follows from the decomposition of the system Hamiltonian according to

$$H = H_e + H_i + V_{ei} \qquad \text{with} \qquad H_a = K_a + V_{aa}\,, \tag{29}$$

and the assumption that the electron-ion term V_{ei} is small compared to the other terms. The collective behavior of the subsystems is accounted for in this approximation, too. However, the influence of the electrons on the ion modes (and vice versa) is neglected.

For the energy transfer between the electron and ion subsystems, one obtains with the FGR approach [13]

$$\frac{\partial}{\partial t}E^{trans}_{e\to i} = -4\hbar\sum_i\int\frac{d\boldsymbol{k}}{(2\pi\hbar)^3}\int_0^{\infty}\frac{d\omega}{2\pi}\,\omega\,\left|\mathrm{V}_{ei}(k)\right|^2\,\operatorname{Im}\chi_{ee}(k\omega)\,\operatorname{Im}\chi_{ii}(k\omega)\left[n^e_B(\omega) - n^i_B(\omega)\right], \tag{30}$$

where $\operatorname{Im}\chi_{aa}(k\omega)$ denote the full density response function of the species a. In RPA, this function is given by

$$\operatorname{Im}\chi_{ee}(k\omega) = \frac{\operatorname{Im}\chi^0_{ee}(k\omega)}{|1 - V_{aa}(k)\chi^0_{ee}(k\omega)|^2}\,. \tag{31}$$

The last relation demonstrates that the response functions $\operatorname{Im}\chi_{aa}$ are sharply peaked at the excitation energies of the collective modes. Therefore, a numerical evaluation of the FGR expression (30) involves the same difficulties as in the case of the coupled mode expression (28). Nevertheless, it can be significantly simplified since the ω-integration in Eq. (30) is effectively limited by the spectrum of the ionic response function $\operatorname{Im}\chi_{ii}(k,\omega)$. This fact justifies the following two approximations that are valid for almost all situations (see also Ref. [25]):

(i) a linearization of the Bose functions, i.e.,

$$n^a_B(\omega/T_a) = (\exp(\hbar\omega/k_BT_a) - 1)^{-1} \approx \frac{k_BT_a}{\hbar\omega}. \tag{32}$$

Obviously, this approximation is applicable as long as the upper limit of the ω-integration is small compared to the thermal energies.

(ii) an evaluation of the electron response function at zero frequency, i.e.,

$$\frac{\operatorname{Im}\chi_{ee}(k,\omega)}{\omega} \approx \left.\frac{\operatorname{Im}\chi_{ee}(k,\omega)}{\omega}\right|_{\omega=0} = \text{const.} \tag{33}$$

Since the ionic response function rapidly decreases at low frequencies (shortly above the ion plasma frequency), the effective range of the ω-integral in Eq. (30) is very small compared to the energy scale of the electron response function. The latter can therefore be used in the low frequency limit.

The remaining ω-integral can now be exactly evaluated using the f-sum rule

$$\int\limits_0^\infty \frac{d\omega}{2\pi}\,\omega\ \operatorname{Im}\chi_{ii}(k,\omega) = \frac{n_ik^2}{4m_i}\,. \tag{34}$$

Since this is an exact relation, no approximation concerning the ion-ion coupling strength has to be made here.

With the upper approximations, one finally gets for the energy transfer rates using the FGR approach

$$\frac{\partial}{\partial t}E^{trans}_{e\to i} = -\sum_i k_B(T_e - T_i)\frac{\hbar n_i}{m_in_e}\int\frac{d\boldsymbol{k}}{(2\pi\hbar)^3}\,k^2\,|\mathrm{V}_{ei}(k)|^2\ \left.\frac{\operatorname{Im}\chi_{ee}(k,\omega)}{\omega}\right|_{\omega=0}. \tag{35}$$

This formula is much easier to evaluate than the original FGR expression since it contains only a single integral over smooth functions instead a double integral over sharply peaked functions.

4.3. Energy Transfer for Classical and Degenerate Plasmas using the FGR Approach

Now the energy transfer rates given by the FGR formula (35) will be investigated in more detail for two important cases: hot, classical plasmas and systems with highly degenerate electrons.

If one considers the plasma electrons as free classical particles, the density response in the low frequency limit is given by

$$\left.\frac{\operatorname{Im}\chi_{ee}(k,\omega)}{\omega}\right|_{\omega=0} = \left.\frac{\operatorname{Im}\chi^0_{ee}(k,\omega)}{\omega}\right|_{\omega=0} = \sqrt{\frac{\pi}{2T_e/m_e}}\frac{n_e}{T_ek}\,. \tag{36}$$

Using this approximation, the k-integral in formula (35) becomes $\int dk\, k^{-1}$, i.e., divergent at the lower and upper boundary. If one uses *ad hoc* cutoffs for the integration limits, an equivalent of the LS formula follows from the FGR approach

$$\frac{\partial}{\partial t} E^{trans}_{e\to i} = -\sum_i k_B(T_e - T_i) \frac{4\sqrt{2\pi} n_i Z_i^4 e^4}{m_i m_e} \left(\frac{m_e}{k_B T_e}\right)^{3/2} \ln\left(\frac{k_{max}}{k_{min}}\right) . \tag{37}$$

The divergencies of the Coulomb integral are a result from the classical treatment and the consideration of free particles (neglect of screening). The generalizations for nondegenerate, weakly coupled plasmas are obvious: the electron response function should be used in RPA and include quantum diffraction effects [24]. In the needed low frequency limit, the electron response is then given by

$$\begin{aligned} \left.\frac{\operatorname{Im}\chi_{ee}(k,\omega)}{\omega}\right|_{\omega=0} &= \left|\operatorname{Re}\varepsilon_e^{RPA}(k,0)\right|^{-2} \left.\frac{\operatorname{Im}\chi^0_{ee}(k,\omega)}{\omega}\right|_{\omega=0} \\ &= \left(\frac{k^2}{k^2+\kappa_D^2}\right)^2 \sqrt{\frac{\pi}{2T_e/m_e}} \frac{n_e}{T_e k} \exp\left(-\frac{1}{8}k^2\lambda^2_{dB}\right) . \end{aligned} \tag{38}$$

Here, $\kappa_D^2 = 4\pi e^2 n_e / k_B T_e$ is the square of the inverse electron Debye screening length, and $\lambda_{dB} = \hbar/\sqrt{m_e k_B T_e}$ is the deBroglie wave length of the electrons. The energy transfer rate that corresponds to this approximation can then be calculated by

$$\begin{aligned} \frac{\partial}{\partial t} E^{trans}_{e\to i} = -\sum_i k_B(T_e - T_i) \frac{4\sqrt{2\pi} n_i Z_i^4 e^4}{m_i m_e} \left(\frac{m_e}{k_B T_e}\right)^{3/2} \\ \times \int_0^\infty dk \frac{k^3}{(k^2+\kappa^2)^2} \exp\left(-\frac{1}{8}k^2\lambda^2_{dB}\right) . \end{aligned} \tag{39}$$

Obviously, no *ad hoc* cutoffs are necessary here. The quantum diffraction and screening result in smooth "cut-offs" at long and short wave lengths, respectively. It turned out that the energy transfer rates corresponding to Eq. (39) coincide with the one of the Boltzmann approach (14) if the transport cross section is used in first Born approximation (see Fig. 2). The reasons for this fact are that the FGR approach is also a first Born approximation concerning the electron-ion interaction and that the FGR formula (39) does not include the ion response function anymore (independent ions give therefore the same results than highly correlated ions).

For systems with degenerate electrons, the upper response functions are not applicable since they neglect exchange effects. If one considers only the response of a system with noninteracting electrons (see Ref. [24] for the response function), the low frequency behavior for any degree of degeneracy needed in Eq. (35) is determined by

$$\left.\frac{\operatorname{Im}\chi^0_{ee}(q,\omega)}{\omega}\right|_{\omega=0} = \frac{m_e^2}{4\pi\hbar^3 k} f_e(k/2,\mu_e) , \tag{40}$$

where $f_e(k,\mu_e)$ denotes the electron (Fermi) distribution with the chemical potential of the electrons μ_e. The consideration of screening on the RPA level is similar to the nondegenerate case, except that the screening length has now to be determined by the relation

$$\kappa_e^2 = 2\frac{e^2 m_e}{\hbar^2 \pi} \sum_{s_z} \int_0^\infty dk f_e(k,\mu_e) , \tag{41}$$

where μ_e is the chemical potential of the electrons. In the limit of nondegenerate systems, this formula yields the classical inverse Debye length, whereas for highly degenerate electrons the inverse of the temperature-independent Thomas-Fermi screening length $\kappa_{TF}^2 = 4e^2 m_e k_F/(\pi\hbar^2)$ follows. The energy transfer rates are then given by

$$\frac{\partial}{\partial t}E_{e\to i}^{trans} = -\sum_i k_B(T_e - T_i)\frac{2}{\pi}\frac{Z_i^2 e^4 n_i m_e^2}{\hbar^3 m_i}\int_0^\infty dk\,\frac{k^3}{(k^2+\kappa_e^2)^2}\,f(k/2,\mu_e)\,. \tag{42}$$

Again we find smooth "cutoff" that ensure a convergent integration. For systems with highly degenerate electrons, the upper cut-off in k-space is now twice the Fermi momentum and the lower cut-off is Thomas-Fermi screening parameter. It should be mentioned that the formula derived here is significantly different than the one given by Brysk [26] which is a product of the LS results and a Fermi distribution.

5. Temperature Relaxation in Nonideal Plasmas

The energy transfer between the electron and ion subsystems discussed in the previous sections describes the evolution of the species temperatures only in the case of ideal plasmas sufficiently. Additionally, the influence of the correlation energies has to be a taken into account for the relaxation of nonideal plasmas (see Sec. 2). Here, the important case of laser-produced plasmas with hot electrons and highly ionized ions will be considered. In this case, contributions from the electron-electron and the electron-ion interaction can be neglected since the kinetic energy of the electrons is much larger. However, the ion-ion interaction can be strong. Therefore, the energy of the ionic subsystem is the sum of the kinetic and the ion-ion correlation energies. The charge state of the ions is here considered to be constant (for the effects of a time-dependent plasma composition see Ref. [5]). The ionic correlation energy is therefore only changing with the ion temperature.

5.1. Correlation Energy for Strongly Coupled Ions

The build up of ionic correlations occurs typically on a time scale of the inverse ion plasma frequency. This is often much faster than the time for the temperature equilibration. The ionic correlations can then be assumed to be instantaneously in equilibrium with the ion temperature.

For the classical ion subsystem, the excess ion-ion correlation energy can be calculated by[3]

$$U_{ii}^{ex} = \frac{n_i^2}{2}\int d\boldsymbol{r}[g_{ii}(r)-1]V_{ii}(r)\,, \tag{43}$$

where V_{ii} is the ion-ion interaction potential and g_{ii} is the ionic pair distribution. In the limit of weakly coupled plasmas, this correlation energy is $U_{ii}^{ex} = \kappa e^2/2$, where $\kappa = (\kappa_e^2 + \kappa_i^2)^{1/2}$ is here the full inverse Debye length that contains electron and ion contributions.

For strongly coupled plasmas, the pair distribution $g_{ii}(r)$ has to be evaluated numerically. Since the build up of ionic correlations is assumed to be instantaneous, an equilibrium treatment can be applied to compute $g_{ii}(r)$. Here, the well-known Ornstein-Zernicke equation with the HNC-closure relation is used [27]. This approach is able to describe the effects of strong coupling and the results are known to agree well with molecular dynamics and Monte Carlo simulations up to coupling parameters of about $\Gamma = 100$ [27].

[3] Here, a term which arises from the neutralizing background, i.e. the electron-ion correlation energy, is included. Since this terms is constant, it does not affect the relaxation.

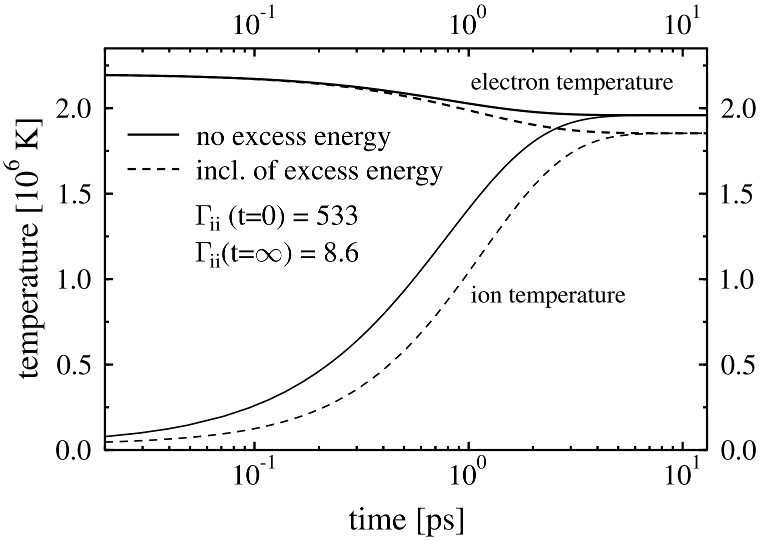

Figure 3. Temperature relaxation in an argon plasma with $n_i = 10^{23}\,\mathrm{cm}^{-3}$ and an ion charge of $Z_i = 8$. The initial electron and ion temperatures are $T_e(0) = 2.2 \times 10^6\,\mathrm{K}$ and $T_i(0) = 3 \times 10^4\,\mathrm{K}$, respectively. For simplicity, the energy transfer rates are calculated with the Landau-Spitzer formula that considers hyperbolic orbits.

5.2. Equilibration of the Species Temperatures

For hot, high-Z materials, the electron temperature changes little during the equilibration of the species temperatures. Changes in the ionic correlation energy due to the time-dependent screening length are therefore negligible. Since we can also neglect the contributions from electron-electron and electron-ion correlations in the considered case, the evolution of the electron and ion temperatures is determined by the set of equations

$$\frac{\partial T_i}{\partial t}\left(\frac{3}{2}n_i k_B + \frac{\partial U_{ii}^{ex}}{\partial T_i}\right) = \frac{\partial}{\partial t}E_{e\to i}^{trans}\,, \tag{44}$$

$$\frac{3}{2}n_e k_B \frac{\partial T_e}{\partial t} = -\frac{\partial}{\partial t}E_{e\to i}^{trans}\,. \tag{45}$$

Fig. 3 shows the solution of these equations for an argon plasma with hot electrons and an ion charge state of eight. For comparison, the figure contains two different calculations: the first neglects the contributions of ionic correlations and shows therefore the relation of an ideal plasma. The second solves the equations (45) and, therefore, contains the ionic correlations. In the given example, the coupling strength in the ionic subsystems changes from $\Gamma_{ii} = 533$ to $\Gamma_{ii} = 8.6$. Related to this decrease of coupling strength is a strong decrease of ionic correlation energy with represents a large sink for the energy that is delivered from the electrons to the ions. As a result, the ions heat much slower (in particular in the beginning of the relaxation) if the ionic correlations are taken into account. This behavior affects also the electron temperature, which is faster decreasing, since the temperature difference and therefore the energy transfer between the species is larger now. Furthermore, the equilibration is slightly slower (about a factor of 1.5) than for the ideal calculation. Due to the different underlying equation of state model in the two calculations (ideal plasma versus Yukawa model), the final common plasma temperature differs significantly, too. Since the ionic correlations become less negative, the full calculation reaches a lower plasma temperature than the ideal one.

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