Local measurement of the plasma diamagnetism
with the Motional Stark Effect on ASDEX

Upgrade

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1 Summary

1.1 Motivation

Fusion is explored for energy production in world-wide effort. To obtain energy in a controlled way from fusion, the most suitable reactions involve fusion of hydrogen isotopes to helium. The main reasons are:

1. The short ranged, strong nuclear force binds the nucleons more tightly together in light elements than in heavier elements. Consequently more energy is released in the fusion reaction of light elements to a heavier one.

2. Lighter elements have a smaller electric charge than heavier elements. Consequently, less energy is required to enable the penetration of the Coulomb barrier.

3. Since the energy loss due to bremsstrahlung scales as a square of the atomic number $Z$, a plasma of a low-$Z$-element loses less energy than a high-$Z$-element plasma.

The reaction, which takes into account these requirements and which requires the lowest temperature, involves deuterium and tritium reacting to form helium and a neutron \[ \frac{2}{1}D + \frac{3}{1}T \rightarrow \frac{4}{2}He (3.5 \text{ MeV}) + \frac{1}{0}n (14.1 \text{ MeV}), \] (1.1)

where the neutrons carry 80\% and the created $He$ particles 20\% of the kinetic energy. This reaction has the highest cross-section, $\sigma_{D-T}^{\text{max}} \approx 5 \cdot 10^{-28} \text{ m}^2$. Moreover the maximal cross-section is reached at the lowest relative energy, $E_{\text{rel}} = 64 \text{ keV}$. For a potential fusion reactor, however, net energy production is only possible, if most of the energy needed for the heating of the plasma is drawn from the fusion reaction itself, i.e. from the 3.5 MeV $\alpha$-particles. The $\alpha$ particles’ energy is envisaged to balance the energy loss, $P_{\text{loss}}$, caused by e.g. radiation and radial transport. The neutrons’ energy is planned to be used as the power plant energy gain and for tritium production by fission of lithium, since tritium is not available on earth in sufficient manner. A simple condition for the net energy can be given by a stationary power balance

\[ \frac{W}{\tau_E} + P_{\text{loss}} = P_{\text{fus}} + P_{\text{ext}}, \] (1.2)

where $P_{\text{fus}}$ is the power produced by the nuclear fusion reactions remaining inside the plasma, $P_{\text{ext}}$ is the power produced by external additional heating devices, $P_{\text{loss}}$ are power losses, e.g., by Bremsstrahlung and $\tau_E$ is the energy confinement time. The thermal energy $W$ stored in the plasma is given by $W = \frac{3}{2}(n_D + n_T + n_e + n_\alpha) k_B T$, where $n_D$ is the deuterium ion density, $n_T$ is the tritium ion density, $n_e$ is the electron density, $n_\alpha$ is the $\alpha$ particles' density and $T$ is the temperature. A condition for a $D - T$ fusion reactor to reach ignition is given by the so-called triple product [2]

\[ n \cdot T \cdot \tau_E > 3 \cdot 10^{21} \frac{keV \cdot s}{m^3}. \] (1.3)
To satisfy this condition, a density \( n \approx 1 \cdot 10^{20} \text{ m}^{-3} \), a temperature \( T \approx 100 \cdot 10^6 \text{ K} \) and an energy confinement time \( \tau_E \approx 2.3 \text{ s} \) are needed.

Two main approaches are subject to present day research to realise fusion power on Earth. In the inertial confinement [3, 4] approach, a \( D-T \)-pellet with a diameter of \( \approx 1 \text{ mm} \) is compressed by a factor of about 1000 and heated up very rapidly to \( T \approx 10 \text{ keV} \) with a high power energy source (laser or heavy ion beams). Under these conditions, inertia is sufficient to confine the plasma for a time duration of \( \approx 1 \text{ ns} \). During this time a considerable fraction of the \( D-T \)-mixture fuses in a controlled explosion. In contrast, the magnetic confinement scheme [5] uses the Lorentz force, which guides the charged particles of a steady state plasma along the magnetic field. The magnetic configuration is of toroidal topology to avoid energy loss in the direction of the magnetic field lines. Furthermore, a poloidal magnetic field component is necessary to overcome the curvature drift, caused by a centrifugal force that is experienced by the particle when following its guiding center orbit along the field lines, and the gradient drift, due to the magnetic field strength decrease from inside to the outside of the torus. The resulting total magnetic field is of helical structure.

In magnetic fusion, two main concepts are pursued: the tokamak [5] and the stellerator [6]. They have in common that their toroidal magnetic field is generated by external currents, but differ in the way the poloidal field is generated. In tokamaks the poloidal magnetic field is produced by a large plasma current. The field line geometry is symmetric with respect to the torus axis, cf. Fig. 1c in Sec. 1.3.1. In stellerators the poloidal magnetic field is generated by external conductors and no plasma current is necessary for the confinement. This leads to a 3-D configuration of the external conductors. In this thesis results from a tokamak are reported.

For stable plasma operation, the plasma needs to be in equilibrium. Within the theory of ideal magneto-hydrodynamics, the stationary force balance can be rephrased to the equilibrium condition \( \nabla p = \vec{j} \times \vec{B} \) [5, 7], where \( \nabla p \) is the pressure gradient, \( \vec{j} \) the plasma current and \( \vec{B} \) the magnetic field.

The possible use of a plasma as energy source requires investigations measuring key parameters in high detail. Being difficult to access but playing a key role at the same time, the magnetic field is required to be measured in this extremely hot matter. As shown by the force balance equation, the magnetic configuration of a magnetically confined plasma is strongly related to plasma properties such as electrical conduction or plasma transport [5]. Moreover, the fusion product, \( \text{He} \), also affects the magnetic confinement by causing diamagnetic effects. It is this entanglement of plasma, fields and fusion processes which make it highly desirable to determine the magnetic fields experimentally. Since these effects lead to small deviations in the magnetic field, e.g. the diamagnetic effect leads to changes in the magnetic field strength by \( \approx 1 \% \) [8, 9], magnetic field measurements of high accuracy are required.

In this Ph.D. project a method is developed to measure the magnetic field and to derive variations in the total plasma pressure due to (dia-) magnetic effects. For this purpose a plasma diagnostic has been set up to measure spectroscopically polarized light. The light is emitted from fast beam-particles excited by the plasma and experiencing the local fields. Since the fast atoms travel through a magnetic field \( \vec{B} \) at velocity \( \vec{v}_b \), a Lorentz field, \( \vec{E}_L = \vec{v}_b \times \vec{B} \), affects the atoms in their moving frame. For this light emission the electric field gives rise to the Motional Stark Effect (MSE). The resulting Stark pattern consists of \( \pi \) and \( \sigma \) lines which are polarized parallel and perpendicular, respectively, to the local Lorentz field. The line splitting, \( \Delta \lambda \), depends on \( |\vec{E}_L| \). Thus, it is possible to conclude from the Stark-spectrum on both, the orientation and the strength of the magnetic field.

Nowadays, multi-channel MSE diagnostics are routinely used as a tool for improved equi-
1.2 Scope and results of the thesis

The purpose of this work is to assess spectral Motional Stark Effect diagnostics for studying plasma effects on magnetic equilibria. Such effects are anticipated due to the plasma diamagnetism and fast-ion pressure effects. The work is split into

1. The successful installation of a spectral MSE diagnostic on ASDEX Upgrade as described in Article I and Article II.

2. A novel approach of data analysis was performed by applying a physics Forward Model for the beam emission spectra. The forward model comprises also the fast ion D\textsubscript{\alpha} signal [26] and the smearing on the CCD-chip. The calculated magnetic field data as well as the revealed (dia)magnetic effects are consistent with the results from equilibrium reconstruction solver. The forward model and a sensitivity studies are presented in Article II.

3. The refinement of the underlying atomic physic: For the very first time the minor atomic effects such as the Zeeman Effect, the correct treatment of the population densities of the upper atomic sub-levels [23, 24, 25], relativistic effects and L-S-coupling are considered in the description of the beam emission spectra. The results showed that these effects significantly changes the magnetic field strength and the direction, e.g. we found that the Zeeman effect and the L-S-coupling affects the magnetic field strength by about 2\%, cf. Manuscript III.

4. The final section shows investigation of fast ion pressure variations and their impact on the local magnetic field in a high $\beta$ discharge scenario. The results agree with calculations from the equilibrium solver (CLISTE) [27] and from the transport code (TRANSF) [28] and shown in the submitted manuscript, Manuscript IV.
1.3 Magnetically confined tokamak plasmas

1.3.1 Magnetic fields in tokamaks

The tokamak [5] is the most advanced magnetic confinement concept in magnetic fusion confinement. The tokamak principle was proposed in the 1950s by A. Sacharov and was studied and improved by L.A. Artsimowitsch in Russia [5]. The tokamak and its magnetic configuration is shown in Fig. 1a. The torus geometry is explained in 1c. The principal magnetic field in a tokamak is the toroidal field $B_\phi$, produced by external coils which surround the toroidal vacuum chamber. The toroidal magnetic field decreases in vacuum radially with $1/R$:

$$B_{\phi,\text{vac}}(R) = \frac{B_{\phi,\text{vac}}(R_0) \cdot R_0}{R},$$

with $R_0$ as the center of the toroidal vessel surrounding the plasma. With the toroidal magnetic field the energy loss along the magnetic field lines is avoided. However, for this geometry two particle drifts towards the plasma rises: the curvature drift, caused by a centrifugal force that is experienced by the particle when following its guiding center orbit along the field lines, and the gradient drift, due to the magnetic field strength decrease from inside to the outside of the torus. These point in the same direction and cause ions to drift upwards and electrons downwards, which at the end results in an electric field perpendicular to the magnetic field lines. To overcome the resulting drifts, a helical structure of the magnetic field lines is required. This structure is achieved by adding a poloidal magnetic field component, $B_\theta$, to the toroidal field component. $B_\theta$ is generated by a strong toroidal current $I_p$ that is induced by a transformer. The resulting magnetic field lines in the tokamak configuration are helical and lie on closed, nested magnetic field surfaces with constant magnetic flux in the form of tori, cf. Fig. 1b. The total magnetic field is given by:

$$B_{\text{total}} = B_{\phi,\text{vac}}(R) + B_{\phi,\text{pol}}(R) + B_{\theta}(R).$$

The definition of the torus coordinates are shown in (c): $R \ldots$ major radius, $r \ldots$ minor radius, $z \ldots$ height, $\phi \ldots$ toroidal angle, $\theta \ldots$ poloidal angle. The major radius of ASDEX Upgrade is $R = 1.65 \text{ m}$. The minor radius of the plasma boundary lies in the range of $a = 0.4 \ldots 0.6 \text{ m}$. Figure 1: (a) Components for magnetic field generation of a tokamak device (picture reproduced from IPP database), (b) magnetic flux surfaces (picture reproduced from IPP database). The major radius of ASDEX Upgrade is $R = 1.65 \text{ m}$. The minor radius of the plasma boundary lies in the range of $a = 0.4 \ldots 0.6 \text{ m}$.
field can be deduced from its poloidal and toroidal components: \( \vec{B} = \vec{B}_\phi + \vec{B}_\theta \) and thus \( |\vec{B}| = \sqrt{|\vec{B}_\phi|^2 + |\vec{B}_\theta|^2} \). From the ratio of \( B_\theta \) and \( B_\phi \) the so-called pitch angle, which is the slope of the magnetic field line on a flux surface relative to the toroidal direction, can be calculated:

\[
\tan \gamma = \frac{|\vec{B}_\theta|}{|\vec{B}_\phi|}.
\] (1.5)

Further coils can be applied for shaping or by changing the position of the plasma, cf. 1a. The induced current heats the plasma resistively and provides the first step towards reaching fusion relevant temperatures. However, this heating method is not sufficient on its own, due to an adverse temperature dependence of the resistivity \( \xi \approx T^{-3/2} \) \([30]\)). Typically additional heating is applied by means of wave heating (ion and electron cyclotron resonance heating, ICRH and ECRH) and neutral beam injection (NBI), which transfers kinetic energy of injected particles (hydrogen or deuterium) to the plasma by collisions.

To characterize the stability of a magnetically confined plasma the so-called safety factor \( q \) is used. In cylindrical geometry \( q \) is a function of the minor radius \( r \) and has the simple form \([5]\):

\[
q = \frac{r B_\phi}{R B_\theta} = \frac{\text{Toroidal winding of one field line}}{\text{Poloidal windings of one field line}}.
\] (1.6)

A big value of \( q \) denote a small inclination of the field lines. For flux surfaces with \( q = m/n \), where \( m \) and \( n \) are integers, the field lines are closed after a few circulations. At these resonant surfaces pressure driven magneto-hydrodynamic plasma instabilities can occur. A condition for the occurrence of these instabilities is given by the non-dimensional \( \beta \), which is the ratio between the mean plasma pressure, \( <p> \), and the magnetic field pressure, \( B^2/2\mu_0 \). For economic reasons \( \beta \) should be as high as possible. From ideal MHD the maximal \( \beta \) can be deduced. Furthermore, the Troyon-limit - a criterion for the onset of pressure driven global MHD stability - sets a different limit on the achievable plasma \( \beta \) in tokamaks. It can be casted to depend on the minor plasma radius, \( a \), the toroidal field and the plasma current, \( I_P \) \([31]\):

\[
\beta = 0.28 \frac{I_P}{a B_\phi}.
\]

### 1.3.2 Fusion experiment ASDEX Upgrade

The experimental results discussed in this thesis were obtained on the ASDEX Upgrade (AUG) fusion device. AUG is a mid-size tokamak located at the Max-Planck-Institute for Plasma Physics in Garching, Germany and went into operation in 1992. Its name is derived from the german ‘Axial Symmetrisches Divertor EXperiment’ which refers to a special magnetic field topology which is also foreseen for ITER. It is europeans’s second largest tokamak device, with a major radius \( R = 1.65 \) m and a minor radius \( a = 0.4 \ldots 0.6 \) m. The toroidal magnetic field, \( B_\phi \), is between 1.9 \ldots 2.8 T and is directed clockwise. It is created by 16 toroidal field coils, each defining one sector of the machine. The poloidal magnetic field is generated by a counter clockwise directed plasma current \( I_P [0.6 \ldots 1.4 \text{MA}] \) that is mainly driven by a central transformer coil. External coils are used to balance the varying magnetic field from the transformer coil and to generate an elongated, triangular shaped magnetic field structure as circular plasmas are less stable versus MHD modes. The inner wall of the experiment is made of tungsten coated graphite tiles which tolerate large
heat loads and which have a high sputtering threshold. The generated plasmas mainly consist of deuterium, as tritium would activate the machine and as hydrogen is less fusion reactor relevant. Plasma densities between $10^{19} \ldots 10^{20}$ electrons per m$^3$ can be achieved and temperatures can reach up to 25 keV. The plasma can be heated externally by Neutral Beam Injection (NBI), Ion Cyclotron Radiative Heating (ICRH) and by Electron Cyclotron Radiative Heating. The maximal heating power input is 33 MW. The duration of plasma discharges is up to 10 seconds, which is a multiple of the global energy confinement time $\tau \sim 100$ ms. The mostly used operational regime is the high confinement mode (H-mode) [32], which typically develops above a certain heating power and which results in a steepening of the edge temperature gradient. Thereby the energy confinement of the plasmas is significantly increased. In the divertor configuration, the outermost flux surfaces are not closed but end in the divertor region. Particles expelled from the plasma do, consequently, not impact on neighboring walls but are guided into the divertor region where they are slowed down by a high density. With the divertor concept, the influx of sputtered wall material is significantly reduced and plasmas with smaller impurity contents are obtained. The plasmas are consequently less cooled by impurity line radiation yielding higher energy confinement times and temperatures.

1.3.3 Plasma equilibrium

1.3.3.1 Ideal magnetohydrodynamics

A magnetically confined fusion plasma can be treated as a fluid. Its macroscopic equilibrium can be determined by ideal magnetohydrodynamics (MHD). The model essentially describes how magnetic force ($\vec{j} \times \vec{B}$), inertial force ($\rho \vec{v} \partial \vec{v}/\partial t$) and pressure force ($\nabla p$) interact with an ideal (i.e., perfectly conducting) plasma, where $\rho$ is the mass density, $\vec{v}$ is the mass flow velocity, $\vec{j}$ is the plasma current density, $\vec{B}$ the magnetic field and $p$ is the plasma pressure which is assumed to be isotropic. It should be noted that in an anisotropic plasma the scalar pressure in Eq. 1.7 has to be replaced by the pressure tensor [33]: $\vec{p} = p_\parallel \vec{1} + (p_\parallel - p_\perp) \vec{e}_\parallel \vec{e}_\perp$, where $p_\parallel$ and $p_\perp$ are the pressure components parallel and perpendicular to the magnetic field respectively, $\vec{1}$ is the unit tensor and $\vec{e}_\parallel$ is the unit vector in the field direction. A detailed treatment for the anisotropic pressure balance is given in [33, 9].

In order to confine a plasma by a magnetic field, a force balance needs to be ensured. The general equation of motion for such a plasma placed in a magnetic field is described by

$$\rho \left( \partial \vec{v}/\partial t + \vec{v} \cdot \nabla \right) \vec{v} = \frac{1}{c} \vec{j} \times \vec{B} - \nabla p.$$  \hspace{1cm} (1.7)

Usually in a fusion plasma the plasma flow velocity is much smaller than $\sqrt{p/\rho}$ (which is of the order of the sound speed in a plasma) and the term $\vec{v} \cdot \nabla \vec{v}$ in Eq. 1.7 vanishes. Consequently in the steady state, where no time variation is involved ($\partial / \partial t = 0$), the fusion plasma can be described by:

$$\nabla p = \vec{j} \times \vec{B}.$$ \hspace{1cm} (1.8)

Although, on a first glance, Eq. 1.8 looks simple, it is not easy to solve the the force balance equation exactly, particularly for toroidal geometries. The problem is multifaceted, e.g. the intrinsically nonlinearity nature of the force balance equation, since the plasma current density and the magnetic field are related through Ampere’s law:

$$\mu_0 \vec{j} = \nabla \times \vec{B},$$ \hspace{1cm} (1.9)
1.3 Magnetically confined tokamak plasmas

where $\mu_0$ is the magnetic constant. Since $\vec{j}$ and $\vec{B}$ are oriented perpendicular to each other, $\nabla p$ must be perpendicular to both $\vec{j}$ and $\vec{B}$:

$$\vec{B} \cdot \nabla p = 0 \quad \text{and} \quad \vec{j} \cdot \nabla p = 0.$$ (1.10)

Eqs. 1.10 allow simple, but important conclusions:

- In ideal MHD on time scales of $\mu s$ there are no pressure gradient along the field lines, thus the pressure is constant on the magnetic flux surfaces.
- Current lines are localized on the magnetic flux surfaces, Grad-Shafranov-Equation.

To solve the balance equation 1.8 for an axisymmetric toroidal system such as the tokamak it can be expressed as a differential equation for the poloidal flux functions $p(\Psi)$ and $F(\Psi)$, with $F(\Psi) = RB_\phi$. The resulting function is the so-called Grad-Shafranov-Equation:

$$\Delta^* \Psi = -\mu_0 R^2 \frac{dp(\Psi)}{d\Psi} - F(\Psi) \frac{dF(\Psi)}{d\Psi},$$ (1.11)

where $p(\Psi)$ is the pressure and $\Psi$ is the poloidal magnetic flux function.

In the here summarized Ph.D. project the interpretive equilibrium reconstruction tool CLISTE [27, 34] was applied to calculate the equilibrium of ASDEX Upgrade discharges relevant for this thesis. The code solves the Grad-Shafranov equation by an iterative approach and uses information from experimental and laboratorial data, e.g. from several magnetic and kinetic measurements.

1.3.3.2 Magnetic effects in the steady state Tokamak plasma

As mentioned in sec. 1.3.1 in tokamaks without a plasma the poloidal magnetic field is zero and according to Eq. 1.4 the vacuum toroidal magnetic field depends on the major radius only. In the presence of the plasma, however, para- and diamagnetic effects cause small deviation from the vacuum toroidal field, e.g. the diamagnetic effect is of about 1%. Thus, for the measurement of the local magnetic field a high accuracy is required.

In the paramagnetic limit toroidal plasma currents induces a poloidal magnetic field. Since the plasma is bound to the magnetic field lines, a poloidal current rises and induces a toroidal magnetic field component that adds to the vacuum magnetic field. In the diamagnetic limit the toroidal magnetic field is decreased by a toroidal magnetic field component which is induced by the gyration of the plasma particles around their guiding center and oriented anti-parallel with respect to the vacuum field. Thus, while the paramagnetic effect is caused by the rising in the toroidal plasma current, the diamagnetic effect is caused by variations of the plasma pressure.

These effects can be quantitatively determined with the balance equation 1.8. For simplicity a cylindrical plasma is assumed, the pressure balance obtained from Eqs. 1.8 and 1.9 results in:

$$\frac{dp}{dr} + \frac{B_\theta}{\mu_0 r} \frac{d(rB_\theta)}{dr} = j_\phi B_\phi,$$ (1.12)
where \( j_\theta = -\frac{1}{\mu_0} \frac{dB_\phi}{dr} \) is the poloidal current density. In the paramagnetic limit, the pressure gradient term is considered to be small compared to the second term in Eq. 1.12:

\[
-\frac{1}{2} \frac{d(B_\theta^2)}{dr} = B_\theta \left( \frac{B_\theta}{r} + \frac{dB_\theta}{dr} \right).
\]

(1.13)

In contrast in the diamagnetic limit the pressure gradient term in Eq. 1.12 is the dominating part and the second term can be neglected, which reduces Eq. 1.12 to

\[
\frac{dr}{d} \left( p + \frac{B_\phi^2}{2\mu_0} \right) = 0.
\]

(1.14)

The effect is quite small \((\delta B_\phi/B_\phi \leq 1\%)\) and high effort is required for its detection.

In tokamaks, both paramagnetic or diamagnetic effects may dominate, depending on the plasma current in relation to the magnetic field. Generally, the diamagnetic effect is a deviation of the paramagnetic limit and not from the vacuum toroidal magnetic field. In Fig. 2 the simulation of a 1\% diamagnetic effect is shown for ASDEX Upgrade relevant parameters, the right plot gives a more detailed view. The vacuum toroidal field is given in black, while the plasma decreases the magnetic field as shown in red. The vertical dashed line indicates the magnetic axis. The poloidal component of \( B \) is mainly affected by toroidal plasma currents, which play a crucial role in the understanding of instabilities (e.g. saw-tooth activity) and transport of the plasma.

1.4 Method of Motional Stark Effect diagnostic

1.4.1 Principle of Motional Stark Effect

The Motional Stark Effect is a method to measure local magnetic fields in hot plasmas. Magnetic field in the interior of high temperature plasmas can be measured by employing the Motional Stark Effect (MSE). MSE is observed on high energy neutrals injected into the plasma. The method was introduced on the tokamaks PBX-M [15] and JET [35] in 1989. The concept relies on the observation of the Balmer-\( \alpha \) transition \((n = 3 \rightarrow 2)\) from high energetic deuterium particles travelling at velocity \( \vec{v}_b \) with respect to the laboratory frame. The beam particles are excited by collisions on ions and electrons, cf. Fig. 3. In the
frame of the moving particles a Lorentz field, $\vec{E}_L = \vec{v}_b \times \vec{B}$, arises. The Balmer-$\alpha$ emission is basically split into 9 detectable Stark components by the electric Lorentz field. The remaining 6 lines have intensities between 0.02% and 0.30% and their contribution can be neglected for all practical purposes. The resulting $\pi$ ($\Delta m_l = 0$) and $\sigma$ ($\Delta m_l = \pm 1$) lines of the Stark pattern are polarized parallel and perpendicular, respectively, to the local Lorentz field. Thus the polarization of the Stark lines is sensitive to the orientation of $E_L$ and allows to measure the orientation of $\vec{B}$. The capability to detect the orientation of $\vec{B}$ is exploited in polarization measurement of the central, unshifted $\sigma$-lines by MSE polarimetry [15, 16, 17]. With spectrally resolved measurements of the Motional Stark spectra the orientation of $\vec{B}$ can be retrieved from the line-ratio of the sum of the $\sigma$ and $\pi$ polarized lines: $\sum I_\pi / \sum I_\sigma$. The observed emission intensity depends on the angle of observation, $\tilde{\theta}$, being the azimuth with respect to $E_L$. Thus, from the anisotropic emission characteristic the orientation of $E_L$ and hence $B$ can be determined:

$$\frac{\sum I_\pi}{\sum I_\sigma} = \frac{\sin \tilde{\theta}}{1 + \cos \tilde{\theta}}.$$  

(1.15)

The line splitting, $\Delta \lambda$, depends on $|E_L|$ and consequently allows to measure $|B|$ [20, 21, 22].

### 1.4.2 Magnetic quantities derived from the MSE spectrum

In table 1.1 relevant spectral quantities of the spectrum, the Stark-splitting and the line intensity are listed. Furthermore, the corresponding parameter to be derived from the data and the equilibrium quantities on which they are sensitive, are given.

Most important fit parameters are the electric Lorentz field and the intensity relation $T_P = \sum I_\pi / I_\sigma$. These quantities are derived from a forward model, which is presented in sec. 1.5.1.

The Lorentz field is calculated from the line splitting of the observed line pattern with the Schwartzschild-Epstein equation. Since the Lorentz field is sensitive to the toroidal magnetic field and to the total pressure, it is a convenient quantity to investigate plasma diamagnetic effects.

The pitch angle determined with the spectral MSE diagnostic is a measure for the direction of the Lorentz field projected in the MSE geometry

$$\gamma_m = \arctan \frac{E_z}{E_x}.$$  

(1.16)

The orientation of $E_L$ is determined by the observation angle $\tilde{\theta}$ and the direction of the
beam. \( \tilde{\theta} \) is a function of the line ratio

\[
\tilde{\theta} = \arccos \sqrt{\frac{1 - T P}{1 + T P}}.
\]

(1.17)

Projected into toroidal coordinates, the pitch angle allows the determining of \( \vec{B}_\phi \) and \( \vec{B}_\theta \).

1.4.3 Experimental setup of the spectral MSE diagnostic at ASDEX Upgrade

The setup of the spectral MSE diagnostic installed at ASDEX Upgrade is shown in Fig. 3 [36, 37]. The sketch in the lower left shows the flux surfaces of the magnetic field configuration. An almost canonical volume reflects the neutral beam injection (NBI) of energetic particles. A fan of lines of sight intersects the NBI and gives the observation channels. Thus signals from different radial positions along the beam axis can be detected. The observed Balmer-\( \alpha \) emission is caused by the plasma beam interaction. After passing a spectrometer the light is detected spectrally resolved by a CCD camera. At ASDEX Upgrade the beam is produced by a positive ion source and injects neutral deuterium at energies of 60 keV, 30 keV and 20 keV with a power of 2.5 MW into the plasma. The beam has a divergence of about 1° and its vertical focal point (at \( R = 2.018 \text{ m} \)) has a cross-section of about 25 cm \( \times \) 27 cm width [38]. For this thesis magnetic fields, \( |B| \), between 2 and 2.6 T are applied.

For radial resolved measurements, the Balmer-\( \alpha \) emission can be observed along the beam axis with an array of 6 \( \times \) 10 (vertical \( \times \) horizontal) lines of sight with varying observation angles. The spectral MSE diagnostic utilizes 6 of 10 horizontal lines of sight to detect the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Principle of the MSE and schematic figure of the spectral MSE set-up at ASDEX Upgrade. NBI = Neutral beam injector, M = mirror, W1 and W2 = cover and vacuum windows, L1, L2 and L3 = lens systems, PEM1 and PEM2 = photoelastic modulators, P = polariser, CCD = charge-coupled device, \( \vec{B} \) = magnetic field, \( \vec{v}_B \) = beam velocity, \( \vec{E}_L \) = Lorentz field, \( \vec{k} \) = direction of line of sight.}
\end{figure}
beam emission. With a mirror (M), which is covered from the plasma by a glass window (W1) the emission is reflected towards the optical path. Since the spectral MSE and the MSE polarimetry employ the same observation system, the beam emission signal also passes through the MSE polarimeter. A lens system (L1) is applied to collect the light, reflected by the mirror, and to guide it through the components of the polarimeter in parallel rays. The polarimeter consists of two photo-elastic modulators (PEM1 and PEM2) and a polariser (P). For each line of sight the collected light is relayed from the torus hall towards the diagnostic hall by a 50 m length optical fibre. A second lens system (L2) is applied to focus the signal onto the fibres (diameter 1 mm), which are arranged in one vertical line at the entrance of a Czerny-Turner spectrometer (focal length 0.75 m, grating of \( g = 1200 \) \( \text{mm}^{-1} \) and F-number 6.5). This allows to acquire both the radial and spectral information of the beam emission with a two-dimensional frame transfer CCD (1024 \( \times \) 1024 with 13 \( \times \) 13 \( \mu \text{m} \) pixels) that is adjusted behind the exit slit. Since the signal was found to be affected by pixel smearing (cross-talk) effects, one channel is covered, (dark reference channel, cf. Fig. 3) to measure this effect. Moreover, one channel is used for wavelength referencing using the spectrum from a neon spectral lamp. Experimentally a small spectral region of about 0.5 nm at \( \lambda = 656.1 \) nm is masked by a blocking wire at the exit slit of the spectrometer to suppress the signal resulting from cold D\( \alpha \) edge emission line. Otherwise this intense emission line would lead to saturation on the CCD-chip. An objective lens system (L3) is used to project the intermediate image of the spectrum and of the blocking wire onto the CCD-chip.

### 1.4.4 Typical observations

A typical spectrum for one position \((R = 1.90 \text{ m}, z = 0.09 \text{ m})\) at \( t = 2.37 \) s is displayed in the upper plot in Fig. 4. Furthermore the corresponding fitted data of a forward model, which is depicted in section 1.5.1 and explained in detail in Article II and Manuscript III is shown. The dominating CX emission line \( \vec{d}_{\text{CX}} \) is slightly shifted with respect to the suppressed cold H\( \alpha \) and D\( \alpha \) emission lines (at around 656.1 nm). The area covered by the blocking wire is indicated by the gray rectangle in Fig. 4. On the red-wing side impurity lines are observed at 657.8 nm and 658.2 nm, e.g. CII \( \vec{d}_{\text{Imp}} \). On the blue-wing side (653 ... 655 nm) a Balmer-\( \alpha \) splitting is clearly visible. It consists of a superposition of three Zeeman, Stark and fine structure (ZMSE) multiplets corresponding to the full, half and third beam energy, each of them Doppler-shifted by \( \Delta \lambda_D \) and overlapping. These are denoted \( \vec{d}_{\text{ZMSE}(E_0)} \), \( \vec{d}_{\text{ZMSE}(E_{1/2})} \), \( \vec{d}_{\text{ZMSE}(E_{1/3})} \). Its characteristic, e.g. Doppler-shift, intensity relation of \( \pi \) and \( \sigma \) lines, \( \sum I_\pi / \sum I_\sigma \), and the splitting, \( \Delta \lambda \), is strongly related to the beam and viewing geometry [37]. Since the spectrum is overlapped partly by the CX emission line and completely by two flat and spectrally broad components (these being the fast ion D\( \alpha \) emission line \( \vec{d}_{\text{FIDA}} \) and the cross-talk on the chip \( \vec{d}_{\text{CT}} \)), a good description of these spectral features is required. However, to pronounce the aforementioned main spectral features, the spectral data are subtracted by a constant background and the cross-talk signal.
1. Summary

Figure 4: Top plot: Experimental data from the ASDEX Upgrade beam emission spectrum $\vec{D}$, modelled spectrum $\vec{d}$, consisting of active and passive charge-exchange emission $\vec{d}_{CX}$, the combined Zeeman and Motional Stark Effect and fine structure multiplets $\vec{d}_{ZMSE}$, CII edge emission $\vec{d}_{Imp}$, fast ion $D_\alpha$ component $\vec{d}_{FIDA}$ and cross-talk $\vec{d}_{CT}$. The filled area represents the calculated ZMSE spectra for the full (blue), half (red) and third (green) energy component. In this measurement the Balmer-α edge emission has been optically blocked to avoid over-exposure of the CCD detector. Both the experimental and the fitted data are background substracted. Bottom plot: $X$ as a measure for the goodness-of-fit.

1.5 Data analysis

The traditional approach to analyze experimental data ($\vec{D}$) is to fit a data descriptive model to those data. Quantities of interest are derived from the best fit to the experimental data. For the case considered in this work, a multi-Gaussian model has been reported to be employed in literature [39]. Given the large number of peaks in the MSE spectrum (cf. Fig. 4), it is evident that the dimensionality of the fit function gets high (amplitudes, widths, shape, background of all peaks). On the other hand, the MSE spectrum is the result of just one quantity of interest affected by a number of settings. Differently to the traditional approach, the concept of integrated data analysis [40, 41] measures the compliance of a model of a measurement with the measured data. The key element of the model of a measurement is a forward function $f$ which is a deterministic prescription to map quantities of interest $\vec{q}$ on modelled data $\vec{d}$:

$$ f : \mathbb{R}^n_q \times \mathbb{R}^n_p \rightarrow \mathbb{R}^n, \vec{d} = f(\vec{q}, \vec{p}). \quad (1.18) $$

Splitting the parameters of $f$ into quantities of interest and model parameters is a choice of convenience; formally all parameters are treated the same way. All quantities are usually
multidimensional. Further model parameters $\vec{p}$ represent settings and are sometimes called nuisance parameters. The forward function may be regarded as a synthetic measurement and is usually called synthetic diagnostics [42]. An interesting application of this view on the forward function allowed on to study effects on the data when settings are changed [40].

In order to link a forward model to the measured data, the measurement error $\vec{\epsilon}$ needs to be taken into account:

$$\vec{D} = \vec{d} + \vec{\epsilon}. \quad (1.19)$$

Finally, the compliance of the forward model with the data is measured by probability distribution functions $P$. Different cases for the statistical description of measurements in terms of specific probability distributions are discussed in literature [43]. For the usual case, that a measurement results in a measured datum, the statistical error of the measurement (standard deviation $\vec{\sigma}$) can be determined

$$\log P \propto \frac{1}{N} \sum \frac{1}{2} \left( \frac{D_i - d_i}{\sigma_i} \right)^2. \quad (1.20)$$

Maximizing $P$ gives the most probable modelled datum which is frequently employed to determine estimates for the parameters. The width of $P$ is used to determine uncertainties of the estimates. It is noted that this formulation can be brought into the broader context of a Bayesian interpretation of probability theory [44, 45] and the discussed case is the limit of the so-called maximum likelihood approach.

Comparing the initially mentioned traditional approach and the forward model based approach practically, the dependencies in the forward model for the MSE case collapses to a few parameters only which are directly retrieved from the quantity of interest. One important reduction of the number of parameters is that physics knowledge is directly used: the magnetic field results in a Lorentz field, both fields give rise to a line splitting. In the traditional line-fitting approach, the splitting is determined from the center of the individual lines. The total number of lines in one MSE multiplet is 15. In the present case, there are three MSE multiplets which are partly overlapping each other. The occurrence of line-broadening leads to additional overlaps and makes the identification of the multiplets contributions very difficult.

Differently, the forward model used here gives the splitting from the fields and atomic physics calculations. In total, five quantities of interests are needed to model the three observed MSE multiplets. For the advanced description of the multiplets, by taking into account minor effects such as Zeeman effect, relativistic effects and the $L$-$S$-coupling the only two additional model parameter are needed. The aforementioned example reflects the concept of Integrated Data Analysis which is a probabilistic modelling of measured data using physics models.

### 1.5.1 Forward model for the full Beam Emission Spectrum

The forward model to describe the measured data $\vec{d}$ consists of a constant background signal ($\vec{d}_{BG}$), carbon impurity lines ($\vec{d}_{Imp}$), active charge exchange ($\vec{d}_{CX}$), a fast ion $D_\alpha$ signal ($\vec{d}_{FIDA}$) and the ZMSE pattern ($\vec{d}_{ZMSE}$). Moreover, cross-talk on the CCD-chip during
readout process ($\vec{d}_{CT}$) is included into the forward model:

$$\vec{d}(F_{EL,B,L-S}, p) = \vec{d}_{CX} + \vec{d}_{Bg} + \vec{d}_{Imp} + \vec{d}_{CT} + \vec{d}_{FIDA} + \vec{d}_{ZMSE} \quad (1.21)$$

The parameter $\vec{p}$ reflects all settings, e.g. calibrations. Within the small range of wavelength the background could be described by a constant. The charge exchange (CX) components (pedestal and active CX emission) were found to be well described by two overlapping Gaussian curves as functions of the wavelength. The widths of the Gaussians can be assigned depending on temperature and rotation velocity, which also effects the shift. At the central channel a temperature of about $T_{CX} \approx 3.5$ keV was determined for the active CX emission line, which is in the range of values determined by kinetic measurements, cf. Fig. 9 in Sec. 1.6. The line is shifted to $\lambda_{CX} \approx 655.83$ nm. The pedestal line has a temperature of about $T_{CX} = 0.3$ keV and is almost unshifted. The amplitudes of the Gaussians are a measure for the number of beam particles.

The Balmer-$\alpha$ splitting is based on a MSE model which is extended by a correction factor that considers the line shift of the MSE lines due to the admixture of the Zeeman effect. The model of the MSE spectrum considers all 15 ($\sigma$ and $\pi$) Stark components with a spectral profile function constructed by a Gaussian. To consider the different energies, three MSE spectra are modelled using the amplitude, $A_{b_i}$, the Doppler-shifted position of the central $\sigma_0$ line, the line position, $\lambda_{E_{Li,\pi,\sigma}}$, and the quantity of interest $T_P = \sum I_{\pi} / \sum I_{\sigma}$

$$\vec{d}_{MSE} = \sum_{i=1}^{3} A_{b_i} \left( I_{\pi} \sum_{\pi} A_{\pi} \exp \left[ -\frac{1}{2} \left( \frac{\lambda - \lambda_{E_{Li,\pi}}}{\sigma_{w_i}} \right)^2 \right] + I_{\sigma} \sum_{\sigma} A_{\sigma} \exp \left[ -\frac{1}{2} \left( \frac{\lambda - \lambda_{E_{Li,\sigma}}}{\sigma_{w_i}} \right)^2 \right] \right). \quad (1.22)$$

The Einstein coefficients $A_{\pi,\sigma}$ for the $\pi$ and $\sigma$ lines of the Stark spectrum are taken from [46]. The beam energy dependent width $\sigma_{w_i}$ is mainly affected by the beam width (Doppler effect) and the instrument function.

In order to take into account the line shift of the MSE lines due to the Zeeman effect and the fine structure, as shown in Sec. 1.5.2, the correction factor $\Delta \lambda_{(E_{L,B})i,\pi,\sigma}$ is implemented into the forward model:

$$\lambda_{(E_{L,B})i,\pi,\sigma} = \lambda_{E_{Li,\pi,\sigma}} + \Delta \lambda_{(E_{L,B})i,\pi,\sigma} \quad (1.23)$$

Thus the improved description of the Balmer-$\alpha$ splitting in the forward model is:

$$\vec{d}_{ZMSE} = \sum_{i=1}^{3} A_{b_i} \left( I_{\pi} \sum_{\pi} A_{\pi} \exp \left[ -\frac{1}{2} \left( \frac{\lambda - (\lambda_{E_{Li,\pi}} + \Delta \lambda_{(E_{L,B})i,\pi,\sigma})}{\sigma_w} \right)^2 \right] + I_{\sigma} \sum_{\sigma} A_{\sigma} \exp \left[ -\frac{1}{2} \left( \frac{\lambda - (\lambda_{E_{Li,\sigma}} + \Delta \lambda_{(E_{L,B})i,\pi,\sigma})}{\sigma_w} \right)^2 \right] \right). \quad (1.24)$$

Non-thermal distribution of sub-levels are considered by a density and beam energy dependent parameter that was calculated by a collisional-radiative model [23] and used as a correction factor for the line ratio of the $\pi$- and $\sigma$-polarized lines

$$T_P^{ns} = c_{ns} \cdot T_P \quad (1.25)$$
Changes in the line ratio and the line mixing effect in the case of the combined Zeeman and Motional Stark effect are considered by an additional correction factor for $T_P$. Thus the corrected line ratio is

$$T_{ns,ZMSE}^{P} = c_T \cdot T_{ns}^{P}.$$ (1.26)

The correction factors for the ZMSE model, $\Delta\lambda_{(E_{L},B)_{\pi,\sigma}}$ and $c_{TP}$, are determined in Manuscript III and their effect is shown in Sec. 1.5.2 and in Manuscript III.

Additional aspects taken into account are non-ideal beam grids which result in focus astigmatism of the beam and the fast ion $D_{\alpha}$ (FIDA) signal. The first aspect causes deviations of the beam direction and width between the three energy components in the applied MSE geometry. This can also be observed in beam-into-gas calibration experiments [47]. Here, no magnetic field is applied during neutral beam injection into a gas. The observed spectrum consists of three Doppler shifted beam emission lines. Each of these lines belongs to a beam energy component and does not overlap with the others, since they are purely Gaussian shaped due to the absence of a magnetic field. Thus separate widths and small deviations in positions can be calculated and incorporated into the forward model for each beam energy component, respectively.

The broad fast ion $D_{\alpha}$ signal overlaps with the whole MSE spectrum but is of low intensity [48]. In order to avoid the high modelling effort required for the small contribution of the FIDA signal, this component is approximated by two overlapping Gaussians of low heights at distinctly different wavelengths and with a large width of $\approx 1.5$ nm (channel dependent), $\vec{d}_\text{FIDA}$.

Although not contributing to the MSE spectrum but for completion of the basic model for the observed spectrum an expression of both impurity carbon lines is given for completion. Here, again sufficient accuracy is achieved when modelling these in a similar fashion to the $D_{\alpha}$-CX lines, using the temperature, mass, line position and amplitude.

Since a frame transfer CCD-camera is used, smearing on the detector is generated during each frame transfer (vertical shift). This adds onto all spectra on the CCD-chip and is considered in the model by $\vec{d}_{CT}$. The smearing between the channels on the CCD-chip is estimated by combining the calibration data obtained from a covered channel, $\vec{d}_{CT0}$, with a channel dependent binning factor, $C_{bin}$ (to gain higher signals several rows are binned to one channel) and the model parameter for smearing, $C_{sm}$: $\vec{d}_{CT} = \vec{d}_{CT0} \cdot C_{bin} \cdot C_{sm}$. The data ($\vec{d}$) on the CCD-chip are given in pixels and had to be mapped onto a wavelength scale. For this purpose three natural lines from a neon spectral lamp were detected on one channel and a quadratic dispersion relation was applied by using the three natural neon lines ($\lambda_{Ne1} = 650.65$ nm, $\lambda_{Ne2} = 653.26$ nm, $\lambda_{Ne3} = 659.87$ nm). Additionally, a channel dependent shift of the wavelength scale due to imperfections of the optics was added [49]. It turns out that the grating of the spectrometer is sensitive to small changes in the ambient temperature. To take into account the resulting variation in dispersion, the dispersion relation is calculated with each plasma discharge.

### 1.5.2 Refinement of the atomic model

Although the influence of the Zeeman Effect onto the Balmer-$\alpha$ beam emission was identified in [50, 24, 39] it was usually neglected in the analysis of this emission. With regard to the MSE$_p$ which analyses the central $\sigma$ line in order to calculate the pitch angle this seemed reasonable [39, 51]. However, when applying the whole spectrum to determine the magnetics, as it is the case for the spectral MSE diagnostic, the impact of minor
effects such as the Zeeman effect, fine-structure and relativistic effects onto the emission has to be considered. In this Ph.D. project it was found that considering these effects in the description of the MSE spectra allows a more accurate estimation of $|\vec{B}|$ and $\vec{B}$ and can lead to improved equilibrium reconstruction. In the following a brief introduction of the combined Zeeman and Motional Stark Effect (ZMSE) will be given. Furthermore, the impact of the Zeeman effect, fine-structure splitting and relativistic effects onto the Balmer-$\alpha$ line will be presented. A detailed investigation of the ZMSE in the Balmer-$\alpha$ beam emission is presented in Manuscript III.

1.5.2.1 Calculation of the combined Zeeman and Motional Stark Effect spectrum

The calculation of atomic data in crossed static electric and magnetic fields was performed in frame of the perturbation theory of the basis of the field-free wavefunctions in the reference frame as shown in Fig. 5. In this coordinate system the Lorentz field $\vec{E}_L = \vec{v} \times \vec{B}$ is taken to be parallel to the $z$-axis and the vector of magnetic field ($\vec{B} = (B, 0, 0)$) is aligned along the $x$-axis. The vector of the velocity $\vec{v}$ is depicted to be in the $x$-$y$ plane ($\vec{v} = (0, -v, 0)$). The direction of observation is shown by the vector $\vec{s}$ with the polar angle $\tilde{\phi}$ and the azimuthal angle $\tilde{\theta}$. The plane normal to the vector $\vec{s}$ defines the direction of the orthogonal polarization vectors $\vec{e}_1$ and $\vec{e}_2$ so that $\vec{e}_1 \cdot \vec{e}_2 = 0$. In addition, we choose the vector $\vec{e}_2$ to be parallel to the $xy$ plane.

The Balmer-$\alpha$ is the transition of hydrogenic atoms from $n = 3$ ($18 |i\rangle$ states) to $n = 2$ ($8 |i\rangle$ states). The aim was to calculate the splitting and the intensity of each transition line in presence of an electric and a magnetic field oriented as shown in Fig. 5. The intensities of the transition lines have a definite spatial characteristic with regard to the polarization and hence depend on the observation angles $\tilde{\theta}$ and $\tilde{\phi}$. The purpose was to calculate the intensities in plane $\perp$ to the line of sight, $\vec{s}$, with regard to the given basis vectors $\vec{e}_1$ and $\vec{e}_2$. The splitting is determined by the transition energy $\Delta E = E_i - E_j$ for the transition $i \rightarrow j$ and was calculated solving the eigenvalue problem, $H\psi = E\psi$, where $\psi$ denotes the hydrogenic wave function and $E$ the eigenvalues. The Hamiltonian $H$ consists of four...
1.5 Data analysis

contributions

\[ H = H^0 + H^{E_L} + H^B + H^{SO}, \]  

(1.27)

The first term of the Hamiltonian is diagonal in spherical basis of the wavefunctions. Except for the energy term of the non-relativist Schrödinger equation it also contains the mass-velocity and Darwin correction. The perturbations caused by Zeeman effect, Motional Stark Effect and the spin-orbit interaction are represented by the terms \( H^B, H^{E_L} \) and \( H^{SO} \), respectively.

We note that the Lamb-shift being on the order of \( 0.0353 \text{ cm}^{-1} \) for \( n = 2 \) levels compared to \( 0.353 \text{ cm}^{-1} \) of fine-structure separation was not included in our calculations. The details of calculations in crossed fields could be found elsewhere \[50\]. In all cases the results reproduced well the cases of pure Zeeman and Stark effects.

Once the eigenvalue problem is solved for Eq. 1.27, the line intensity can be calculated. The emission characteristic of the polarized transition lines depends on the observation angles \( \tilde{\phi} \) and \( \tilde{\theta} \), cf. Fig. 5. Hence, in case of a statistical distribution among the sub-levels the intensity of a multiplet component in the directions of \( \vec{e}_1 \) and \( \vec{e}_2 \) is

\[ I^{(ij)}_{(1,2)} = |D^{(ij)}_{(1,2)}|^2 \]  

(1.28)

with the transition probability \( D^{(ij)}_{(1,2)} = \langle i|e_{(1,2)} \cdot r|j \rangle \). Here, the dipole approximation is used to calculate the matrix elements of \( \mathbf{D}^{(ij)}_{(1,2)} \) considering their emission characteristic into the direction \( \vec{e}_1 \) and \( \vec{e}_2 \). The viewing geometry can be extracted from the left hand side of Fig. 5.

As shown in Manuscript III, the line pattern obtained for the case of the combined Zeeman and Motional Stark effect differs from the pattern obtained for the pure MSE case. The line splitting is not equidistant any more and shifted towards higher wavelengths. Furthermore, the intensities of the \( \pi \) - and \( \sigma \) -polarized lines change, e.g. the pure Stark \( \pi \) transitions obtain the small fraction of the \( \sigma \) contribution and on the other hand the pure Stark \( \sigma \) transitions obtain certain fraction of \( \pi \) components. In all cases the sum over all \( \sigma \) and \( \pi \) components remains constant, though the different polarizations appear at the same positions compared to the Stark effect. This mixing effect is of the order of 1 - 3 %, cf. Manuscript III. While the mixing of different states gives apparently only a minor correction, it is noted that these spurious contributions (change in the line splitting and line ratio) may lead to systematic deviations relevant to, e.g., small anticipated effects due to potential fast-ion pressure.

In the upper sub-figure of Fig. 6 the modelled Doppler-shifted emission patterns are shown for given experimental conditions for both calculations, MSE and ZMSE, normalized to their maximum value. ASDEX Upgrade relevant beam energies \( E_0 = 29.8 \text{ keV/amu} \), \( E_{1/2} = 14.9 \text{ keV/amu} \), \( E_{1/3} = 9.95 \text{ keV/amu} \) form a pattern represented by the blue, red and green curves. The MSE results are plotted in solid lines; the ZMSE results are shown in dashed lines. The ZMSE pattern is plotted in yellow and only slightly deviates from the MSE pattern (black). To reveal the spectral differences between both models, the difference \( I_{ZMSE} - I_{MSE} \) is plotted in the lower sub-figure in Fig. 6. The obtained difference between both models up to 4.0 % with respect to the maximum intensity. It is noted that the observed difference is strongly related to the chosen geometry setting (\( \vec{E}_L, \vec{B} \) and \( \vec{s} \), cf. Fig. 5). For observation of the emission along \( \vec{E}_L \) (\( \tilde{\theta} = \pi \)) all polarization directions perpendicular to \( \vec{E}_L \) will be observed, (\( \pi_B, \sigma_B \) and \( \sigma_{E_L} \)). At line-of-sight parallel to \( \vec{B} \) (\( \tilde{\theta} = \pi, \tilde{\phi} = \pi \)), all multiplet components which are perpendicularly polarized to \( \vec{B} \) are observable (\( \sigma_B, \sigma_{E_L} \) and \( \pi_{E_L} \)).
1.5.2.2 Impact of the combined Zeeman and Motional Stark effect to the derived quantities

In the forward model a correction factor \( c_{TP} \) was introduced that considers changes in the line ratio due to the Zeeman effect, cf. Sec. 1.5.1. For MSE geometry and beam energies at ASDEX Upgrade the correction factor is \( c_{TP}(E_0, E_{1/2}, E_{1/3}) \approx \{0.98, 0.97, 0.96\} \). These values vary for different channels, due to the different observation angles and magnetic field. Fig. 7a shows how the physics quantity, \( \gamma \), is affected by the admixture of \( \vec{B} \). The MSE angles calculated from the spectral MSE (black) and spectral ZMSE data for three ASDEX Upgrade beam energies (blue, red, green) with varying \( T_P \) are presented. The thin black lines represent a typical \( (T_P, \gamma) \)-relation as found in the measurements applying the MSE forward model and are used for the discussion below. For the three beam energies the same \( T_P \) value leads to the corrected MSE angles indicated by the vertical dashed lines. The effect on the MSE angle measurement of \( \Delta \gamma = \{0.31^\circ, 0.52^\circ, 0.74^\circ\} \) is quite significant compared to the required accuracy for fusion devices which is in the range of 0.1°...0.5° [22]. A second finding of the MSE angle analysis is that the correction increases linear with the applied beam energy but increases slightly non-linear with \( T_P \). It can be concluded that MSE angle reconstructions suffer systematically from a neglect of the Zeeman effect and its correction is in the order of about 0.7°. As mentioned in Sec. 1.5.1, the Zeeman effect and the fine-structure cause a shift of the multiplet and a change in the line splitting. For ASDEX Upgrade relevant conditions the multiplet is shifted by about 5% for 30 keV/a.m.u. to 11% for 30 keV/a.m.u. beam energies with respect to the \( \sigma_0 \)-Stark line. The line splitting changes in the range of 1% (30 keV/a.m.u) to 2% (30 keV/a.m.u.). In Fig. 7b the change in \( |\vec{B}| \) due to the difference of the line splitting between pure MSE case and ZMSE case is shown for varying splitting and ASDEX Upgrade beam energies. The splitting is the mean value taken from most intensive lines \((-4\pi \ldots +4\pi)\). The scattered symbols denote...
1.6 Measurements

1.6.1 Reconstruction of a linear magnetic field ramp-down phase

In order to validate the forward model, a well diagnosed reference discharge has been conducted on ASDEX Upgrade. The discharge parameters were chosen to reflect conditions which have been analysed with the CLISTE equilibrium code [34, 27] independently. For a stationary plasma current of \( I_p = 0.8 \text{ MA} \) and an applied heating power of \( P = 5.8 \text{ MW} \) the toroidal magnetic field was systematically decreased from \( |B_\phi| = 2.6 \text{ T} \) to \( |B_\phi| = 2.4 \text{ T} \), cf. Fig. 8a. If the forward model is correct, then the temporal evolution of the Lorentz field \( \vec{E}_L = \vec{v} \times \vec{B} \)

- Should show the same variation as the applied magnetic field

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**Figure 7:** (a) Pitch angle variation due to changes in line ratio for three different ASDEX Upgrade beam energies. The thin black lines show a typical \((T_p, \gamma)\)-relation when applying the MSE forward model. The dashed lines mark the \(\gamma\) value for the same \(T_p\) value but calculated with the ZMSE forward model. The colors indicate the certain ASDEX Upgrade beam energy. (b) Magnetic field variation as a function of the line splitting at the radial position \( R = 1.94 \text{ m} \). The crosses represent MSE (along the solid lines) and ZMSE (along the dashed-dotted lines) calculated splittings corresponding to a magnetic field ramp performed during ASDEX Upgrade discharge 26322. The lines along the experimental data represent a fit to these data. The horizontal black line indicates a magnetic field calculated with CLISTE. The vertical lines show the corresponding splitting value calculated with the MSE model (solid lines) and with the ZMSE model (dashed-dotted lines). The data are represented color-coded for the three beam energies full (black), half (blue) and third (red).
1. Summary

- Should agree with the independent analysis of CLISTE equilibrium code

Fig. 8b shows the time traces of the Lorentz field $\vec{E}_L = \vec{v} \times \vec{B}$ from both an independent analysis of CLISTE (blue line) and from the fitted data of the forward model (red line) for two chosen channels.

The linear ramp-down phase between $t = 3.8$ s and $t = 6.2$ s was assumed to follow the linear decrease of $B_\phi$ and therefore fitted by a linear model. The precision for each channel was estimated from the sum of the squared residuals. The resulting 2$\sigma$ error intervals are represented by the shaded regions and are about the same order for CLISTE and forward model data. However, in contrast to the CLISTE data, the precision of the forward model data was found to be channel dependent. With $\sigma = 0.3\%$ the error is the lowest at the outermost channel and rises towards the plasma core with a maximum value of $\sigma = 0.6\%$ for the innermost channel. This can be explained by the beam attenuation which leads to a decreasing signal-to-noise level towards the plasma.

![Figure 8: Reference discharge on ASDEX Upgrade (#26322): (a) Time traces of the toroidal magnetic field (red) and the plasma current (black). (b) Time traces of the Lorentz field calculated with the CLISTE equilibrium code (blue) and with the ZMSE forward model (red): For both methods the fit functions (straight lines) and the related 2$\sigma$ confidence intervals (shadowed regions) are given.](image-url)

The results show a small radius dependent difference in the off-set and a good agreement for the temporal variation between both methods. The deviation in the bias is channel dependent up to 2.5\%. The deviations are - besides errors in the equilibrium reconstruction - due to systematic errors, mainly imperfections in the optics components in the spectral MSE set-up, such as non-optimal adjustment of the detection components (spectrometer, objectives and CCD-chip). Another error source is the use of an improper spectral profile function for the MSE lines. In the present work a Gaussian profile was applied. However, Dux et al. [47] has shown that the MSE profile is asymmetric due to the variation of the magnetic field along the line-of-sight when it is crossing a beam with a certain width. The effect is the strongest for the innermost channel.

In contrast to the bias the inclination is not affected by systematic errors. A small maximal deviation between $-1\%$ and $-9\%$ for the applied linear decrease of approximately 6\% in the magnetic field was found, resulting in a total error in the variation of the Lorentz field of $\Delta m/E_{L0} \approx 0.5\%$.

All in all, it can be concluded that with the given precision and accuracy local variations in the magnetic fields can be described with an accuracy less then 0.5\%. The bias has to be minimized to improve the consistency with CLISTE data. This could be done by decreasing or removing the aforementioned systematic errors, e.g. applying asymmetric MSE profile
functions. However, the findings show that the application of the combined Zeeman and Motional Stark effect forward model is a suitable tool to confirm and, moreover, to improve equilibrium reconstructions.

1.6.2 Fast ion effects in NBI heated high $\beta$-discharge

1.6.2.1 Discharge overview

In order to assess the potential sensitivity of spectral combined Motional Stark effect measurements to fast ion effects, a discharge with stepwise increasing heating power up to 10.8 MW and with a total toroidal plasma current of $I_p = 0.8\,\text{MA}$ during the flat-top phase ($t > 0.8\,\text{s}$) was conducted within this work. An external toroidal magnetic field of $B_\phi = -2.48\,\text{T}$ was applied. Fig. 9 shows relevant time traces of discharge# 26323 on ASDEX Upgrade. Fig. 9a indicates the applied heating: the electron cyclotron heating (ECRH) was applied in order to prevent tungsten accumulation in the plasma center [52, 53, 54]. Neutral beam injection (NBI) heating with deuterium beams was provided by four 2.5 MW NBI sources for $t > 1.2\,\text{s}$.

Fig. 9b shows waveforms reflecting the plasma response. The pressure is derived from the experimental data $p_{\text{kin}} = k_B \cdot (n_e T_e + n_i T_i)$. The fast ion pressure, $p_{FI}$, has been determined with the transport code TRANSP. The magneto-hydrodynamic pressure is the total pressure, $p_{\text{mhd}} = p_{\text{kin}} + p_{FI}$. Furthermore, the stored fast ion and magneto-hydrodynamic energies, calculated with TRANSP are given in Fig. 9b as well.

The time evolution of the Lorentz fields calculated with the ZMSE forward model is shown in Fig. 9c for one channel (black line). The data are compared to results of the CLISTE code (blue line). Here, CLISTE was constrained by magnetic measurements by the pick-up-coils outside the plasma, the safety factor (cf. Eq. 1.6) on the magnetic axis ($q_{\text{ax}} = 1$) and by the total pressure profile.

The main aspects are the stepwise increase and decrease of the NBI heating power at time points indicated by the vertical dotted lines. The time traces of the central total pressure and central total energy reflect the heating pattern: additional NBI heating leads to a rise and reduced NBI heating leads to a decrease of both quantities. The diamagnetic decrease in the magnetic field due to the rise in the total pressure can be observed in the decrease of the modelled Lorentz field in Fig. 9c. As shown in Eq. 1.14 the diamagnetic decrease is related to changes in the toroidal magnetic field.

As depicted in Fig. 9b, additional NBI heating not only increases the thermal plasma pressure but also the production of high energetic particles (fast ions), which gyrate around their guiding center and thus induce a magnetic field component almost anti-parallel to the toroidal magnetic field. The high contribution of the fast ion pressure in the total pressure of more than 30% indicates that the generated fast ions lead to detectable changes in the magnetic configuration and need to be considered in equilibrium reconstruction. This effect is reduced for lower total pressures.

As aforementioned, both, the combined Zeeman and Motional Stark effect as well as the CLISTE calculated Lorentz fields show a significant response on the heating pattern. The stepwise increase and decrease of the NBI heating power lead to a change in $|\vec{E}_L|$ followed by an exponential decay phase. In order to calculate the Lorentz field variation due to changes in the heating scenario the CLISTE and forward model data were fitted with an exponential decay:

$$E(t) = E_0 + \Delta E \left( 1 - \exp \left( -\frac{t_0 - t}{\tau_D} \right) \right),$$

(1.29)


1. Summary

![Image of graph](image)

**Figure 9:** ASDEX Upgrade discharge 26323: Time traces of the heating power (a) and the central kinetic, mhd and fast ion pressure as well as the related mhd and fast ion energies (b). The time evolution of the Lorentz field calculated with the ZMSE forward model (black) and with CLISTE (blue) are shown in panel (c).

with the fit parameter $E_0$ denoting the Lorentz field at the beginning of each heating phase, $\Delta E$ denoting the amplitude of the change of the Lorentz field and $\tau_D$ the decay time. The latter fit parameter is a measure for the confinement times in ASDEX Upgrade. The obtained values differ in a range of 20 ms...160 ms with a high uncertainty of about 50 ms due to the high noise and low time resolution in the data. However, these times comply in magnitude with the known slowing down times of fast ions and with the energy confinement time for the ASDEX Upgrade, which are about 60 ms. $t_0$ represents the onset-time of each heating scenario phase. All four parameters are dependent on the heating interval and on the position ($R, z$). Investigations in Manuscript III have shown that the forward modelled data have a lower noise level for the outer channels than CLISTE data. In contrast to the CLISTE data the noise rises towards the plasma core due to the beam attenuation. The shaded area indicates the $1\sigma$ interval of confidence of the fit. It was found a channel dependent deviation of 0.45% (Ch4)...1% (Ch5) with a mean deviation of $\text{rms} = 0.7\%$. All findings and comparison of modelling with experiment indicate good agreement. Correspondingly, both models show a similar response on the heating variation in the calculated Lorentz field.

1.6.2.2 Fast ion pressure variation deduced from the forward modelled Lorentz field variation

From the related Lorentz field variation the total pressure variation can be deduced using the pressure balance equation in cylindrical approximation, cf. Eq. 1.12. In the diamagnetic limit, where the pressure gradient $\Delta p$ is the dominating part, Eq. 1.12 can be written as
Manuscript IV:

\[ \Delta p \approx -\frac{\Delta E_{\text{Dna}}}{E_L} \cdot \frac{B^2}{\mu_0}. \]  

(1.30)

Here the Lorentz field variation is assumed to be about the toroidal field variation. \( \Delta E_{\text{Dna}} \) describes variations in the Lorentz field due to the diamagnetic effect, without field variations caused by the shift of the magnetic axis, \( \Delta E_{\text{SL}} \). The latter one is about \( \Delta E_{\text{SL}}/E_L \leq 0.1\% \) for the present discharge.

**Figure 10:** Comparison of pressure profile variations for the heating scenario transition NBI5 switched on (a) and (c) and NBI5 switched off (b) and (d): the upper panels show the total and kinetic pressure variation, the lower panels present fast ion pressure variation. Error bars from error propagation equation taking into account the 1\( \sigma \) uncertainty of \( \Delta E_L, E_L, \) and \( B \).

Fig. 10a and 10b show the profiles of the pressure variations when NBI5 is switched on (10a) and off (10b). Results from different methods TRANSPI (mhd), kinetic measurements (kin), forward model (ZMSE) and CLISTE (CL) are compared with each other. Consistent to the findings in Fig. 9b, additional heating power leads to a rise and reduced heating to a decrease of the total pressure and kinetic pressure. The effect of the heating is most significant in the plasma center, here \( |\Delta p_{\text{tot}}| \approx 40 \text{kPa} \) and \( |\Delta p_{\text{kin}}| \approx 23 \text{kPa} \) when NBI5 is
switched on. Towards the plasma edge the pressure variation vanishes. This indicates that the pressure profile gradient increases with additional NBI heating and vice versa. The pressure profile gradient calculated from the forward model data shows the same behaviour. In fact, within the errors, the forward modelled data (black bold line) show a good agreement with the total pressure results from TRANSP (black dashed line) for both cases, NBI5 on and NBI5 off. Moreover, these results are consistent with the determination of the total pressure variation by CLISTE. In the errorbars, the channel and time dependent uncertainties of $\Delta E_L$, $E_L$ and of $B$ are included.

With the knowledge of the kinetic pressure change the fast ion pressure variation can be calculated. The results (black line with symbols) are compared with the TRANSP calculations (red dashed lines) in the sub-figures 10c and 10d for the transitions NBI5 on and NBI5 off. Although there are discrepancies of about $1 \cdots 5$ kPa, the profiles are consistent in their respective shape; the data fit within their $1\sigma$ confidence interval.

These results indicate the developed method and analysis approach to be sufficiently sensitivity for the detection of fast-ion effects in plasma equilibria.

1.7 Conclusions

In this Ph.D. project local diamagnetic effects due to changes of the local total pressure of a hot fusion plasma have been experimentally investigated and have been compared with modelling. The effects are small ($\leq 1\%$) and thereby pose so far unattained requirements on the measurement. A diagnostic technique of combining both, high accuracy and precision with a sophisticated data analysis has been developed. For this purpose the spectral Motional Stark effect diagnostic was revisited and applied on ASDEX Upgrade (IPP Garching, Munich). An improved data analysis was applied on the experimental data of the Balmer-$\alpha$ emission to derive the strength and direction of the local magnetics. In contrast to previous works we not only considered the Motional Stark effect, but also minor effects such as the Zeeman effect, the fine-structure and relativistic effects when analysing the beam emission spectrum. With the improved data analysis we were able to detect small diamagnetic effects in a high-$\beta$ discharge.

Typically, the data analysis was performed by fitting the experimental data to a multi-Gaussian model with a large set of fit parameters [33]. Here instead, we fitted the data to a forward model which leads to a significant reduction of the amount of parameters. Moreover, since the forward model parameters are accessible to experimental error assessments, they are easily employed for the application of constraints. The data analysis procedure improves the goodness-of-fit to a level which allowed small changes to be detectable.

In the forward model the main spectral features are considered: charge exchange, impurity carbon lines, fast ion $D_\alpha$, smearing of the data on the detection device and the MSE. Moreover, Zeeman effect, fine-structure and non-statistical distribution of upper atomic sub-levels [23] are included into the atomic package of the forward model. The former effects were already pointed out in earlier works [24, 50] but were neglected in the analysis of hydrogenic beam emission spectra in hot fusion plasmas. Since under ASDEX Upgrade conditions the effect of radial electric fields is predominated by the combined Zeeman and Motional Stark effect, they are not considered in this thesis. However, the package is designed in a way that additional atomic features could be easily implemented.

The contribution of the Zeeman effect and fine-structure to the Balmer-$\alpha$ emission spectrum was systematically investigated for different beam energies, magnetic field strengths and geometries. It was found that under typical ASDEX Upgrade conditions the combined Zeeman and Motional Stark effect affects the line splitting and line ratio, leading to...
significant changes in the derived magnetic field strength and pitch angle. The magnetic field strength differs about 2.2% with respect to the value calculated from the pure MSE case (referred to the half beam energy component), which is in the range of the para- and diamagnetism. The pitch angle changes about 0.7° with respect to values based on the atomic models of pure Stark effect. This is significantly higher than the required accuracy for fusion devices which is in the range of 0.1° to 0.5°. These findings clearly show that the Zeeman effect has to be included into the atomic model in order to determine the local magnetic field with high accuracy from the beam emission pattern. On the other hand, the central σ line of the emission pattern is almost unaffected by the Zeeman effect. Therefore, its contribution can be neglected in MSE polarimetry measurements which utilizes this line to determine the pitch angle.

The assumption of non-statistical distribution of the upper sub-levels leads to even higher changes in the pitch angle. Calculations have shown a shift up to 3° and show that this effect need to be considered in the atomic model, too.

To validate the ZMSE forward model, the beam emission spectra of a discharge with an applied magnetic field decrease was analysed. The data showed a good agreement with results from the equilibrium reconstruction solver CLISTE. The difference in the inclination of the time development in the Lorentz field strength was \( \Delta \left( \frac{\delta E_L}{E_L} \right) \leq 0.5 \). The achieved accuracy was \( \Delta E_L / E_L \leq 2.5 \% \) with a precision of \( \leq 0.2 \% \). The high accuracy in both, the absolute value and the time development demonstrates the spectral MSE diagnostic with the ZMSE forward model to be a tool for accurate equilibrium reconstruction. The error estimated from the statistical noise is slightly lower then the error of the CLISTE data for the outer channels but increases towards the inner channels due to beam attenuation.

The forward model was applied to a high-β discharge to determine variations in the total and fast ion pressure due to stepwise increasing and decreasing neutral beam injection heating power (up to \( P_{NBImax} = 10 \text{ MW} \)). A rise of the local diamagnetic effect due to additional NBI heating power could be observed in the Lorentz field. The related rise in the fast ion pressure population was derived from the increase in the Lorentz field. The changes of about 0 kPa at the plasma edge to 15 kPa at the plasma center are consistent with results from calculations performed with the transport code TRANSP and with the equilibrium solver CLISTE.

A worthwhile continuation of our work would be the reduction of noise by improved hardware settings, e.g. using an individual optical port. Less uncertainty of the signal could be achieved by an improved geometry, which leads to a bigger Doppler shift and thus to less overlapping of the main spectral features, charge exchange and combined Zeeman and Motional Stark effect. Furthermore, a full statistical treatment of the forward model parameters would possibly lead to a further reduction of the uncertainty. Higher accuracy also might be achieved by a more detailed description of the atomic physics involved, e.g. considering the effect of radial electric fields on the particles. Unwanted and unknown polarization effects, e.g. by the plasma wall should be described. Finally, with a faster CCD-camera fast plasma instability processes can be investigated.

In this thesis a spectral Motional Stark effect diagnostic was designed on ASDEX Upgrade for the local measurement of magnetic configuration. An improved accuracy could be achieved by the approach of an forward model for data analysis. We extended the typically applied atomic description of the MSE multiplet by demonstrating that the Zeeman effect and the fine-structure significantly affect the Balmer-α splitting. For the very first time we could show that these minor effects have an significant impact on the derived magnetic field. Moreover, we could confirm the crucial effect of non-statistical distribution of the upper sub-levels on the pitch-angle. We were able to detect small diamagnetic
effects in a high-$\beta$ discharge. The results are consistent with calculations from transport and equilibrium codes. From results of this thesis it can be concluded that the spectral MSE diagnostic is a tool for the local measurement of the plasma diamagnetism. Hence the present thesis provides essential information for an revised measurement of the magnetic equilibrium in magnetically confined fusion plasmas and therefore can significantly contribute to an improved confinement of the plasma.
2 Bibliography


[54] H. Höhnle, W. Kasparek, J. Stober et al., Extension of the ECRH operational space with O2 and X3 heating schemes to control W accumulation in ASDEX Upgrade, Technical report, ASDEX Upgrade Team (2010). 21
3 List of Figures

1. (a) Components for magnetic field generation of a tokamak device (picture reproduced from IPP database), (b) magnetic flux surfaces (picture reproduced from IPP database). The definition of the torus coordinates are shown in (c): R...major radius, r...minor radius, z...height, φ...toroidal angle, θ...poloidal angle. The major radius of ASDEX Upgrade is $R = 1.65$ m. The minor radius of the plasma boundary lies in the range of $a = 0.4...0.6$ m.

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5. Frame of reference and vectors orientation used in the present calculation: $\vec{B}$ is the vector of magnetic field, $\vec{E}_{L}$ is the vector of induced Lorentz field, $\vec{v}_{L}$ is the vector of atom velocity, $\vec{s}$ denotes the direction of observation, $\vec{e}_{1}, \vec{e}_{2}$ are polarization vectors, $\phi$ and $\theta$ are the angles determining the observation orientation. The electric field induces linear polarized emission in the direction parallel to $\vec{E}_{L}$ ($\pi_{E_{L}}$), circular polarized emission perpendicular to $\vec{E}_{L}$ ($\sigma_{E_{L}}$); the magnetic field induces linear polarized emission in the direction parallel to $\vec{B}$ ($\pi_{B}$) and circular polarized emission perpendicular to $\vec{B}$ ($\sigma_{B}$).

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7 (a) Pitch angle variation due to changes in line ratio for three different ASDEX Upgrade beam energies. The thin black lines show a typical \((T_P, \gamma)\)-relation when applying the MSE forward model. The dashed lines mark the \(\gamma\) value for the same \(T_P\) value but calculated with the ZMSE forward model. The colors indicate the certain ASDEX Upgrade beam energy. (b) Magnetic field variation as a function of the line splitting at the radial position \(R = 1.94\, m\). The crosses represent MSE (along the solid lines) and ZMSE (along the dashed-dotted lines) calculated splittings corresponding to a magnetic field ramp performed during ASDEX Upgrade discharge 26322. The lines along the experimental data represent a fit to these data. The horizontal black line indicates a magnetic field calculated with \textit{CLISTE}. The vertical lines show the corresponding splitting value calculated with the MSE model (solid lines) and with the ZMSE model (dashed-dotted lines). The data are represented color-coded for the three beam energies full (black), half (blue) and third (red).

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9 ASDEX Upgrade discharge 26323: Time traces of the heating power (a) and the central kinetic, mhd and fast ion pressure as well as the related mhd and fast ion energies (b). The time evolution of the Lorentz field calculated with the ZMSE forward model (black) and with \textit{CLISTE} (blue) are shown in panel (c).

10 Comparison of pressure profile variations for the heating scenario transition NBI5 switched on (a) and (c) and NBI5 switched off (b) and (d): the upper panels show the total and kinetic pressure variation, the lower panels present fast ion pressure variation. Error bars from error propagation equation taking into account the 1\(\sigma\) uncertainty of \(\Delta E_L, E_L\) and \(B\).
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5 Thesis Articles

Authors contributions:
The ASDEX Upgrade Team conducted the experiments and conducted the analysis of data relevant to the characterization of the discharge.

I Motional Stark Effect Spectra Simulations for Wendelstein 7-X
R. Reimer designed and realized the experimental set-up spectral MSE diagnostic and performed the measurements on ASDEX Upgrade; R. Reimer and A. Dinklage developed the forward model for the analysis of beam emission spectra; R. Reimer performed the data analysis of experimental data of an ASDEX Upgrade discharge; R. Reimer and A. Dinklage wrote the manuscript. It was edited by all the co-authors (A. Dinklage, J. Geiger, J. Hobirk, M. Reich, R. Wolf). The ASDEX Upgrade discharge was performed by J. Hobirk.

II Spectrally resolved motional Stark effect measurements on ASDEX Upgrade
R. Reimer improved the MSE hardware. L. Sawyer and R. Reimer improved the MSE forward model. R. Reimer characterized and validated the MSE forward model. R. Reimer performed the data analysis and wrote of the manuscript. The final editing involved all the co-authors (A. Dinklage, R. Fischer, J. Hobirk, T. Löhhard, A. Mlynek, M. Reich, L. Sawyer, R. Wolf).

III On the influence of Zeeman effect on magnetic equilibrium reconstruction using Motional Stark Effect diagnostic
R. Reimer, A. Dinklage, R. Wolf, O. Marchuk, M. Dunne, B. Geiger, J. Hobirk, W7-X Team and ASDEX Upgrade Team
Submitted in Plasma Physics and Controlled Fusion
R. Reimer developed the MSE angle calculations on ASDEX Upgrade from beam emission spectra. R. Reimer and O. Marchuck development the improved atomic package of the spectral MSE forward model. Experimental work and data analysis was done by R. Reimer. R. Reimer and O. Marchuck wrote the manuscript. It was edited by all the co-authors (A. Dinklage, R. Wolf, O. Marchuk, M. Dunne, B. Geiger, J. Hobirk). The ASDEX Upgrade discharge was operated by J. Hobirk. M. Dunne performed equilibrium reconstuctions of ASDEX Upgrade discharge.

IV Combined Zeeman and Motional Stark Effect measurements of local magnetic effects on ASDEX Upgrade
Mc Carthy, W7-X Team and ASDEX Upgrade Team

Submitted in Nuclear Fusion

R. Reimer developed MSE angle calculations on ASDEX Upgrade from beam emission spectra, performed experimental work and data analysis and wrote the manuscript. It was edited by all the co-authors (A. Dinklage, R. Wolf, O. Marchuk, M. Dunne, B. Geiger, J. Hobirk, P.J. Mc Carthy). Transport calculations were conducted by B. Geiger. The ASDEX Upgrade discharge was operated by J. Hobirk. M. Dunne and P.J. Mc Carthy performed equilibrium reconstructions of ASDEX Upgrade discharge.

V Forward modelling of Motional Stark Effect spectra

A. Dinklage, R. Reimer, R. Wolf, Wendelstein 7-X Team, M. Reich, and ASDEX Upgrade Team, Fusion Science and Technology 59, (2011)

R. Reimer and A. Dinklage developed the MSE forward model. A. Dinklage designed the synthetic diagnostic for Wendelstein 7-X. R. Reimer performed data analysis of experimental ASDEX Upgrade data. A. Dinklage wrote the manuscript. It was edited by R. Reimer and R. Wolf.

VI Motional Stark Effect Measurements of the Local Magnetic Field in High Temperature Fusion Plasmas


R. Reimer contributed to the paper with experimental work using the spectral MSE set-up and spectral MSE measurements on ASDEX Upgrade. Under the supervision of A. Dinklage he developed an improved atomic model, including the Zeeman effect in the spectral MSE forward model. O. Ford developed the Imaging MSE diagnostic on ASDEX Upgrade and performed first IMSE measurements on ASDEX Upgrade. This method is based on a diagnostic development of J. Howard. As supervisor of the PhD thesis of A. Bock, J. Stober contributed with helpful discussions about equilibrium results and issues of MSE polarization measurements. A. Bock contributed with measurements of the influence of the plasma density on the MSE polarization measurement, suggesting an influence of the plasma background radiation on the polarization measurement. The original MSE polarization measurement at ASDEX Upgrade was designed and implemented by R. Wolf and later further developed by J. Hobirk and M. Reich. R. Wolf developed an early MSE diagnostic on JET. He wrote the manuscript. The final editing involved all the co-authors (A. Bock, O. Ford, R. Reimer, A. Burckhart, A. Dinklage, J. Hobirk, J. Howard, M. Reich, and J. Stober).

Confirmed by: Greifswald, 10. August 2016

(PD. Andreas Dinklage) (René Reimer)
5.1 Article I

Motional Stark Effect Spectra Simulations for Wendelstein 7-X
R. Reimer, A. Dinklage, J. Geiger, J. Hobirk, M. Reich, R. Wolf, ASDEX Upgrade, and Wendelstein 7-X Teams

Motional Stark Effect Spectra Simulations for Wendelstein 7-X

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Key words Motional Stark effect, plasma diamagnetism, diagnostic simulation.

Simulations of motional Stark effect (MSE) spectra for Wendelstein 7-X (W7-X) are reported. While contributing to experimental equilibrium reconstruction in general, the measurements aim at the detection of small deviations of vacuum fields, such as the diamagnetic effect. For the diagnostic beam on W7-X, the expected plasma $\beta$ implies a diamagnetic effect as low as 3%. A data model for the measurement has been developed for the application to W7-X. The model includes polarisation effects and the beam attenuation to estimate the required signal dynamics. The model has been applied to ASDEX Upgrade data.

1 Introduction

Motional Stark effect measurements have been shown to give access to magnetic field properties both in tokamaks and stellarators [1, 2]. In this paper, the capabilities to determine small deviations from vacuum fields such as diamagnetic effects ($\Delta \beta/\beta \propto \Delta B/B$) by spectrally resolved MSE spectroscopy are addressed. In contrast to usual MSE polarimetry [3, 4], the full spectrum is measured to determine local effects on $\vec{B}$ [5].

The observed spectrum consists of the combination of the emission from the deuterium plasma and the beam. This complex structure of the Balmer-$\alpha$ transition can be divided into three main parts. The cold edge feature with an underlying broad pedestal, also present before the neutral beam is switched on, the charge exchange (CX) signal and beam emission. In general, with viewing lines inclined to the direction of an injected beam the beam emission is Doppler shifted and thus can be distinguished from the edge and CX emission. Therefore, the three energy fractions of the beam (full, half and third energy components) cause the beam emission to divide into three partially overlapping parts of varying intensity. In addition to that, the Doppler shifted emission lines from the moving beam particles show a Stark multiplet structure due to the electric Lorentz field $\vec{E}_L = \vec{v}_B \times \vec{B}$ in the moving particles rest frame.

The magnetic field $\vec{B}$ can be evaluated by exploiting the following two aspects: 1) Using the knowledge of the spectral separation between the Stark components and considering that only the projection $B_\perp$ of $\vec{B}$ on the plane perpendicular the beam causes $\vec{E}_L$ [4], the absolute value of the magnetic field $B_\perp$ can be derived. Provided changes of the splitting are most sensitive to changes of the toroidal magnetic field, direct information about the plasma diamagnetism can be obtained. 2) Via the ratio of the $\pi$ and $\sigma$ components of the Stark multiplet, the polarization characteristics of the detected light:

$$\frac{\sum I^\pi}{\sum I^\sigma} = \frac{\sin^2 \gamma \sum A_\pi n_\pi}{1 + \cos^2 \gamma \sum A_\sigma n_\sigma},$$

and thus the projection $B_\perp$ can be derived. $\gamma$ is the angle between $\vec{E}$ and line of sight, $n_\pi$ and $n_\sigma$ are the upper state populations.

The paper deals with two main objectives. First, MSE spectra are simulated for measurements on Wendelstein 7-X. The analysis and the simulations are linked: key-part of the simulation - a data model - can be applied for the
analysis of the experimental data within a Bayesian approach [8]. Therefore, measurements of spectrally resolved MSE diagnostic currently implemented on ASDEX Upgrade are analyzed. Thus, the analysis is employed to validate the simulation model.

2 Simulation of the Motional Stark effect on Wendelstein 7-X

This section describes the forward model to simulate the MSE spectra for W7-X. In addition to its application for simulation, the forward model will also be used for data analysis by fitting the model to experimental data. A full simulation needs to include noise and further uncertainties appropriately [6]. The full physical description of the data is the goal of the Integrated Data Analysis [7, 8].

How much details a forward model needs, depends on the data or, respectively, on the simulation application. For these first simulations of MSE spectra on Wendelstein 7-X, some emphasis was put on simulating the dynamic range. Consequently, the charge exchange background contained in the MSE spectrum (cf. Fig. 5) has been addressed in some more detail. A flow chart summarizing the main simulation elements is shown in Eq. 2.

\[
\begin{align*}
\text{plasma profiles} & \quad \downarrow \quad v_{\text{beam}} \quad \downarrow \quad \text{beam-attenuation} \quad \downarrow \quad \text{optics} \quad \downarrow \quad d\vec{I}(\lambda, \text{pol}, \vec{x}) \quad \rightarrow \quad \text{signal} \\
\vec{B} & \rightarrow \vec{E}_L \rightarrow \vec{E} \rightarrow \text{geometry}
\end{align*}
\]

As central input, the magnetic field \( \vec{B} \) has been taken from VMEC simulations for different cases of predictive transport simulations. One group refers to systematic variations of the plasma-beta, a second one for evolving bootstrap current and a third simulation refers to a bootstrap current compensation by electron cyclotron current drive. The latter cases have been simulated as described in Ref. [9]. The resulting Lorentz field components in W7-X cartesian coordinates are shown in Fig. 1 for a specific case.

![Flow chart summarizing the main simulation elements](image)

Plasma profiles \((n_e, n_i, T_e, T_i)\) from transport simulations are used to determine the beam attenuation by calculating the rate coefficients of a collisional radiative model (Fig. 2 (a)). Losses from neutral beam states are ionization and proton collision induced charge exchange. The resulting neutral hydrogen in the plasma is ionized again both by electron and proton collisions. The model does not include any diffusion of charge exchange neutrals in the plasma at present (beam-halo).
For first simulations, the collisional-radiative beam plasma model has been applied for the envisaged 60-keV-\(H^0\) diagnostic beam. As shown in Fig. 2 (a) the collisional radiative model yields the densities of the \(n=3\) excited states both for the neutral beam and the charge exchange neutrals in the observation volume. Atomic data were taken from Refs. [10, 11]. The model is applied on the full, half and third energy components of the beam; the specified beam current fraction of these three components (70%:15%:15%) is accounted for in the signal fluxes described below. The simulation results for the beam attenuation as well as for the density distributions along the beam axis are shown in Fig. 2 (b).

![Diagram](image1)

**Fig. 2** Graphical representation of the collisional radiative model used for the forward model (a) and results for the case of bootstrap current compensation at \(t = 5\) s of the simulation (b). The abcissa is a coordinate along the central beam-line through the plasma, corresponding to the radius (starting at the beam source). (Color figure: www.cpp-journal.org).

The Stark splitting has been determined with the Schwarzschild-Epstein formula. Considering the Doppler shift of the lines, taking into account the excited state atoms profiles, assuming a statistical distribution of excited state atoms in the \(n=3\) state, the emission from the beam can be determined. For the simulation, the line broadening of the beam emission was assumed to be due to the beam divergence (specified to be 1 degree) and for the charge exchange line to result from the ion temperature. The divergence of the line-of-sight has not been included so far. Fig. 3 (a) shows the spectral emission density in photons per second as emitted from the observed volume; the subplot (b) shows the spectral emission densities at the assumed end of the line-of-sight, i.e. the polarized fractions have been corrected for their emission anisotropy. For this simulation, the plasma profiles and the VMEC equilibria for the bootstrap-current compensation (after 4s compensations) were taken [9].

![Diagram](image2)

**Fig. 3** Results of emission densities in the \(H_\alpha\) spectral range. For the simulation, 15 channels along the RuDI beam-line were considered, each channel corresponding to one color. Channel \#3 is the first with non-zero emission. The peaks corresponding to different observation volumes move from the blue-shifted lower channel numbers to the red-shifted higher channel numbers. (a) shows the emission in total; (b) is corrected for the azimuthal emission anisotropy of polarized components. (Online colour: www.cpp-journal.org).

As a first model for the observation optics, the polarization transmittance of optical elements have been modeled. Examples of differing optics have been simulated before the detection of the light. For this purpose, the
Stokes vector of the emitted light has been calculated. The effect of polarizers has been taken into account by their respective Müller-matrices [12].

3 Measurements of motional Stark effect spectra on ASDEX Upgrade

To validate the simulation model, the forward model has been applied to ASDEX Upgrade data. Fig. 4 shows the experimental setup for MSE spectroscopy installed at ASDEX Upgrade [13]. Here, data from a field ramp ($B_{\text{tor}}(r=1.65\,\text{m})=2.475\,\text{T} \rightarrow 2.65\,\text{T}$) are analyzed. MSE spectra were taken from a 60 keV D-beam. For the wavelength calibration, the emission of a neon spectral lamp was used.

The MSE observation mainly consists of optical fibers, a Czerny Turner spectrometer and a CCD-camera. A wire, appropriately positioned in the focal plane of the spectrometer, is used to suppress the intensive edge emission line to prevent saturation of the CCD-chip. A lens system is inserted for imaging the intermediate image with the wire onto the chip.

![Fig. 4 Schematic view-graph of the experimental setup of MSE spectra measurement on ASDEX Upgrade. (Color figure: www.cpp-journal.org).](image)

![Fig. 5 (a) MSE spectrum from ASDEX Upgrade (shot 25827) (data: bold lines). The corresponding modeled data of the forward model are displayed in dashed lines. (b) Shows the $\chi^2$ for each pixel. Impurity lines C II and calibration wavelengths (Ne I) as indicated. (Color figure: www.cpp-journal.org).](image)

For a first estimation of the measurement capabilities of the spectrally resolved MSE on ASDEX Upgrade measurement, MSE spectra at the beginning and at the end of a B-field ramp were analyzed. Fig. 5 (a) shows data from pulse 25827 at two times of the discharge. The spectra consist of the MSE multiplet from the beam blue shifted from CX $D_\alpha$ emission from the plasma. The spectral shift of the MSE lines reflects the Doppler effect. The effect of the above mentioned wire can be seen in the very center of the broad $D_\alpha$ emission. This leads to an almost complete suppression of the much narrower, cold $D_\alpha$ emission ($\lambda=656.1\,\text{nm}$) line from the plasma edge. The remainder of the spectrum consists of impurity radiation (C II at about 80 eV) and a Bremsstrahlung background. Also experimental data for the background without diagnostic beam are displayed in Fig. 5 (a) (NBI 3-off). Additionally Fig. 5 (a) shows a fit of the forward model to the data. The MSE emission could be modeled with Lorentz fields as indicated in Fig. 5. The ratio of the Lorentz fields matches with the ratio of toroidal fields within 1%. The variation of B ($\delta B/B \approx 7\%$) can be easily resolved. The Doppler shift of the MSE emission corresponds to the beam particle velocity and the viewing angle. The width of the MSE components reflects the beam divergence. Also a correction due to the polarization anisotropy is included into the fitted model. Deviation from a non-uniform population of the excited sub-levels needed not to be taken into account. In addition to the CX emission from the heating beam, the $D_\alpha$ emission has been modeled by two CX components ($t=4.0\,\text{s}$: $T=1\,\text{keV}$ and $2.5\,\text{keV}$, $t=5.5\,\text{s}$: $T=0.6\,\text{keV}$ and $\cdots$).
1.6 keV) which had to be slightly Doppler shifted to consider the plasma rotation. The edge emission has not been included since the experimental setup suppresses this narrow contribution. Furthermore, the C II impurity lines were fitted. Fig. 5 (b) shows $\chi^2 = ((f - d)/\sigma)^2$ for each pixel, where $f$ are the values from the model and $d$ are the data. The error (CCD noise) $\sigma$ was determined from independent calibration measurements and found to be intensity dependent. The normalized $\chi^2/N$ for the fitted models is about 20. From Fig. 5 (b) systematic deviations of the model from the data can be seen in the spectral range of 651 to 653 nm and 658 to 659.5 nm. This could reflect $D_\alpha$ CX emission from fast ions [14]. The periodic structure at the wavelength range of the MSE emission could be due to non-Gaussian profile shapes of the multiplet lines, reflecting a non-Gaussian beam. The Ne I emission results from a small cross talk of an adjustment channel for wavelength calibration.

4 Summary

A forward model for MSE spectra has been developed. The model was used to simulate MSE spectra for W7-X parameter. The forward model has also been applied to experimental data. A fit of the model gives excellent reconstruction of data. However, systematic features - such as broad emission beside the MSE spectrum and beam profile - need further investigation.

The simulations of W7-X current drive experiments indicate the effect on the Stark splitting for the diagnostic beam to be virtually independent from details of the discharge scenario (cf. Fig. 1) also in cases for large variations of the global rotational transform. This reflects the dependence of the rotational transform on the large aspect ratio of Wendelstein 7-X. A full variation of the plasma beta expected for W7-X gives in the chosen configuration a variation of the Lorentz field of the order of 3% in the plasma center. This complies reasonably with the detection capabilities from the experimental spectra. Moreover, polarization simulations indicate sensitivity of the detected multiplets’ amplitudes polarizing elements and the emission anisotropy denoting the relevance of polarization ($\pi/\sigma$).

The encouraging analysis results indicate effort for a refinement of the forward model to be reasonable. The simulation needs to cover different observation geometries as well as the option to employ the neutral beam injection heating beams. A consideration of finite observation waists and a finite extent of the beams is required for sensitivity assessments. A fundamental issue in the interpretation of Stark spectra is also the quantification of non-statistical upper level populations in the observed multiplets due to details in the beam excitation [15], which was not found for the analyzed ASDEX Upgrade spectra. Finally, appropriate estimates of noise sources as well as a more realistic observation optics need consideration for a quantitative assessment of measuring capabilities on Wendelstein 7-X.

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References

5.2 Article II

Spectrally resolved motional Stark effect measurements on ASDEX Upgrade
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Spectrally resolved motional Stark effect measurements on ASDEX Upgrade

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A spectrally resolved Motional Stark Effect (MSE) diagnostic has been installed at ASDEX Upgrade. The MSE data have been fitted by a forward model providing access to information about the magnetic field in the plasma interior [R. Reimer, A. Dinklage, J. Geiger et al., Contrib. Plasma Phys. 50, 731–735 (2010)]. The forward model for the beam emission spectra comprises also the fast ion Dn signal [W. W. Heidbrink and G. J. Sadler, Nucl. Fusion 34, 535–615 (1994)] and the smearing on the CCD-chip. The calculated magnetic field data as well as the revealed (dia) magnetic effects are consistent with the results from equilibrium reconstruction solver. Measurements of the direction of the magnetic field are affected by unknown and varying polarization effects in the observation. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4829665]

I. INTRODUCTION

The reconstruction of magnetic equilibria is crucial for the analysis of diagnostic results in magnetic confinement fusion experiments. In tokamaks, equilibrium calculations based on the solution of the Grad-Shafranov equation can be effectively constrained by measurements.¹,² Examples for such experimental input for equilibrium reconstruction include magnetic flux measurements outside the plasma and Faraday rotation measurements³ yielding information from the plasma interior.

An active diagnostic measuring local components of the field employs the Motional Stark Effect (MSE) of the emission from fast injected particles excited by the hot plasma.⁴,⁵ Due to the linear field dependence of the Motional Stark-splitting, MSE measurements are routinely performed on the Dn-line of excited beam particles.

The Stark-splitting and thus | | from the Stark-effect can be determined by applying the Epstein-Schwarzschild formula:

\[ \Delta E_n(eV) = 7.94198 \times 10^{-11} \times |E_L|(Vm^{-1})n(n_1 - n_2), \]

where \( n \) denotes the principal quantum number and \( n_1 \) and \( n_2 \) are parabolic quantum numbers.⁹

Contributions resulting from the Zeeman effect were neglected. This is reasonable for the Dn beam emission with beam energies of the order of 30 keV/amu at the present geometry with a minimal angle between beam and magnetic field line of \( \alpha_{min} \approx 55^\circ \). Under these conditions the relation between the energy splittings resulting from the Zeeman effect \( (\gamma/\Delta) \) and resulting from the Stark effect \( (\epsilon/St) \) is

\[ \gamma/\Delta = \frac{\mu_B}{(3/2)\alpha_{min}} \frac{1}{v_B \sin(\alpha_{min})} \]

\[ \epsilon/St = 0.0036, \]

a)See authors list of A. Kallenbach et al., Nucl. Fusion 51(9), 094012 (2011).

II. PHYSICS OF MSE

The Stark-splitting and thus \( |B| \) can be determined by

\[ \frac{\gamma}{\Delta} \text{ or } \frac{\epsilon}{\text{St}}, \text{ depending on the orientation of } \alpha_{min}. \]

Under these conditions the relation between the energy splittings resulting from the Zeeman effect \( (\gamma/\Delta) \) and resulting from the Stark effect \( (\epsilon/St) \) is
with the Bohr magneton, $\mu_B$, the Bohr radius, $a_0$, the elementary charge, $e$, and the beam velocity $v_B$.

The direction of the magnetic field is reflected by the Stark line ratio $T_p$ of the $\sigma$ and $\pi$ components due to their emission characteristics:

$$T_p = \frac{\sum I_{\pi i}}{\sum I_{\sigma i}}$$  \hspace{1cm} (4)

$$\approx \frac{\sin^2 \theta}{1 + \cos^2 \theta}$$  \hspace{1cm} (5)

with $\theta$ being the azimuth with respect to $\vec{E}$.

In order to take into account the non-statistical distributions of the Stark sub-levels a correction factor, $c_{ns}$, is introduced:

$$T_{ns} = \frac{T_p}{c_{ns}}.$$  

$c_{ns}$ depends mainly on the plasma density, magnetic field, and injected beam energy.

### III. EXPERIMENTAL IMPLEMENTATION

#### A. Experimental set-up

Figure 1 shows the experimental set-up of the sMSE diagnostic at ASDEX Upgrade (major radius $R = 1.65$ m, minor radius $r = 0.5$ m).

The maximum magnetic field is $B = 3.1$ T. For the sMSE a magnetic field of about 2 to 2.6 T is applied. The beam emission comes from a 2.5 MW heating beam. The beam, produced by a positive ion source injects neutral deuterium at energies of 60 keV, 30 keV, and 20 keV into the plasma and has a divergence of about 1°. At its vertical focal point (at $R = 2.018$ m) the beam has a cross-section of 25 cm $\times$ 27 cm width.

The $D_{\alpha}$ emission, caused by the plasma beam interaction, can be observed by an array of $6 \times 10$ (vertical $\times$ horizontal) lines of sight (LOS) with varying observation angles. Thus, it is possible to detect signals along the beam axis at different radial positions. The sMSE diagnostic utilizes six of ten horizontal LOS.

For this prototypical study, the detection optics of the MSE polarimeter were shared. As shown in Fig. 1, the LOSs intersect a mirror (M), covered by a glass window (W1). Since the sMSE and the MSE polarimetry employ the same observation system, the beam emission signal also passes through the MSE polarimeter. A lens system (L1) collects the light to guide it through the components of the polarimeter (photo-elastic modulators, PEM1 and PEM2 and polarizer, P) in parallel rays.

For each LOS the collected light is relayed from the torus hall towards the diagnostic hall by a 50 m length optical fibre. A lens system (L2) is applied to focus the signal onto the fibres. Since the fibres (diameter 1 mm) are arranged in one vertical line at the entrance of a Czerny-Turner spectrometer (focal length 0.75 m, grating of $g = 1200$ mm$^{-1}$, and F-number 6.5), a two-dimensional frame transfer CCD (1024 $\times$ 1024 with 13 $\mu$m pixels) is used behind the exit slit to collect both the radial and spectral information from the beam emission.

Since the signal was found to be affected by pixel smearing (cross-talk) effects, one channel is covered (dark reference channel cf. Fig. 1) to measure this effect. Moreover, one channel is used for wavelength referencing using the spectrum from a neon spectral lamp. Experimentally, a small spectral region of about 0.5 nm at $\lambda = 656.1$ nm is masked by a blocking wire at the exit slit of the spectrometer to suppress
the signal resulting from cold D_e edge emission line. In other case this intense emission line would lead to saturation on the CCD-chip. The intermediate image of the spectrum and blocking wire is projected onto the CCD-chip with a lens system (L3).

B. Experimental results

A typical spectrum and fitted data (as explained in Sec. IV) for one position (R = 1.90 m, z = 0.09 m) at t = 2.37 s are displayed in the upper plot in Fig. 2. The dominating feature is the CX emission line (d_{CX}) which is slightly shifted with respect to the suppressed cold H_e and D_e emission lines (at around 656.1 nm). The area covered by the blocking wire is indicated by the grey rectangle in Fig. 2. On the red-wing side impurity lines are observed at 657.8 nm and 658.2 nm, e.g., CII (d_{Imp}).

On the blue-wing side (653...655 nm), the MSE spectrum consisting of three Stark multiplets (corresponding to the full, half, and third beam energy) each Doppler-shifted and overlapping is clearly visible. These are denoted d_{MSE(E0)}, d_{MSE(E1/2)}, d_{MSE(E1/3)}. The fact that the MSE spectrum is overlapped partly by the CX emission line and completely by two flat and spectrally broad components (these being the fast ion D_e (FIDA) emission line d_{FIDA} and the cross-talk on the chip d_{CT}) stresses the importance of a good description of these spectral features. The remarkably high quality of the fit is reflected in the lower plot by the goodness-of-fit. The error analysis, as well as the applied forward model will be outlined in detail in Sec. IV.

IV. DATA ANALYSIS

A typical approach to analyze the experimental data (D) is to fit the modelled data (d) to a multi-Gaussian model with a large set of fit parameters. Here instead, the data are fitted to a forward model. Practically, the forward model has much less parameters than the multi-Gaussian model. Furthermore,
the forward model parameters are accessible to experimental error assessments and are easily employed for the application of constraints.

A flow-chart of the forward model is shown in Fig. 3. Based on the physical cause, namely, the magnetic field, and the beam energy, as well as the beam observation geometry, the signal detected on the CCD-chip is simulated. The magnetic field can be used as a constraint for equilibrium calculations\(^{16}\) (an example equilibrium, calculated with the CLISTE equilibrium solver, is given at the left-hand side of the flow-chart). A typical sample of the detected experimental data is shown by the right plot of the flow-chart. The color coded signal of the beam emission, detected by the CCD-chip, is plotted channel and spectrally resolved.

The forward modelled data have been fitted to the experimental data, by optimizing the model parameters. The fit is based on the minimization of \(\chi^2\) where, \(\chi^2 = \sum_i(D_i - d_i)^2/\sigma_i^2\) applying the Nelder-Mead simplex algorithm. \((D_i - d_i)\) denotes the residual \(\epsilon\) of the fit at the \(i\)th pixel. The pixel and channel dependent error \(\sigma_i\) of the data has been determined by calibration measurements at varying radii. The goodness-of-fit per pixel, \(\chi^2\), in the lower subplot of Fig. 2 shows that the forward model gives a very accurate description of the experimental data leading to normalized values of \(\chi^2 = 2.3\) (reduced by the number of points to be fitted: \(\chi^2 = 1/N \sum_i(D_i - d_i)^2/\sigma_i^2\)).

A. Forward model of MSE spectrum

1. Basic model (df\(^3\))

Following the experimental observation in Fig. 2 the data on the detector \((D)\) can be described by Eq. (6) and is composed of the following contributions: the Stark spectrum \((d_{\text{MSE}})\),\(^{13,14}\) the active CX line \((d_{\text{CX}})\), a continuous background, e.g., Bremsstrahlung radiation, \((d_{\text{BG}})\), and impurity lines \((d_{\text{imp}})\). The emission from the cold plasma edge is suppressed by the aforementioned blocking wire and does not need to be considered here:

\[
df^3 = d_{\text{MSE}} + d_{\text{CX}} + d_{\text{BG}} + d_{\text{imp}}. \tag{6}
\]

The model of the Stark spectrum considers all 15 (\(\sigma\) and \(\pi\)) Stark components. The spectral profile function is constructed by a Gaussian. Due to the occurrence of beam particles of three different energies, three Stark spectra are modelled using the amplitude, \(A_b\), line position, \(\lambda_{\text{MSE},\sigma}\), and the quantity of interest \(T_{\sigma} = T_{\pi}/T_{\sigma}\):

\[
d_{\text{MSE}} = \sum_{i=1}^{3} A_b \sum_{\sigma} A_{\sigma} \exp \left[ -\frac{1}{2} \left( \frac{\lambda - \lambda_{\text{MSE},\sigma}}{\sigma_{\text{MSE}}} \right)^2 \right] + T_{\pi} \sum_{\sigma} A_{\sigma} \exp \left[ -\frac{1}{2} \left( \frac{\lambda - \lambda_{\text{MSE},\sigma}}{\sigma_{\text{MSE}}} \right)^2 \right]. \tag{7}
\]

The Einstein coefficients \(A_{\pi,\sigma}\) for the \(\sigma\) and \(\pi\) lines of the Stark spectrum are taken from Ref. 10. The width \(\sigma_{\text{MSE}}\) is mainly affected by the beam width (Doppler effect) and the instrument function. The correction of non-statistical distribution of the Stark sub-levels is made in a later step by introducing the correction factor, \(c_{\text{MSE}}\), as explained in Sec. II.

The background and CX components (pedestal and active CX emission) were found to be well described by a constant \(\left(A_{\text{BG}}\right)\) and two overlapping Gaussians as functions of the wavelength \(\lambda\):

\[
d_{\text{CX}}(\lambda) = \sum_{i=1}^{2} A_{\text{CX}} \sqrt{\frac{m_{\text{CX}, c^2}}{2 \pi k_B T_{\text{CX}, \lambda_{\text{CX}}}}} \times \exp \left[ -\frac{1}{2} \frac{m_{\text{CX}, c^2}}{k_B T_{\text{CX}, \lambda_{\text{CX}}}} \left( \frac{\lambda - \lambda_{\text{CX}}}{\lambda_{\text{CX}}} \right)^2 \right]. \tag{8}
\]

\[
d_{\text{BG}} = A_{\text{BG}}. \tag{9}
\]

with \(k_B\) being the Boltzmann constant and \(c\) the speed of light. The widths of the Gaussians can be assigned depending on temperature and rotation velocity, which also affects the shift, reflected by \(\lambda_{\text{CX}}\). At the central channel a temperature of about \(T_{\text{CX}} \approx 3.5\) keV was determined for the active CX emission line, which is in the range of values determined by kinetic measurements, cf. Fig. 7. The line is shifted to \(\lambda_{\text{CX}} \approx 655.83\) nm. The pedestal line has a temperature of about \(T_{\text{CX}} = 0.3\) keV and is almost unshifted. \(A_{\text{CX}}\) denotes the respective amplitudes as a measure for the number of particles. \(m_{\text{CX}}\) represents the atomic mass of the CX particle (deuteron).

Although not contributing to the MSE spectrum but for completion of the basic model for the observed spectrum an expression of both impurity carbon lines is given for completion. Here, again sufficient accuracy is achieved when modelling these in a similar fashion to the \(D_\alpha\)-CX lines, using the temperature, mass, line position, and amplitude \((T_{\text{imp}}, m_{\text{imp}}, \ldots)\).
\( \lambda_{\text{Imp}, i}, A_{\text{Imp}, i} \):

\[
d_{\text{Imp}} = \sum_{i=1}^{4} A_{\text{Imp}, i} \sqrt{\frac{m_{\text{Imp}, i} c^2}{2 \pi k_B T_{\text{Imp}, i} \lambda_{\text{Imp}, i}^2}} \times \exp \left[ -\frac{1}{2} \frac{m_{\text{Imp}, i} c^2}{k_B T_{\text{Imp}, i}} \left( \frac{\lambda - \lambda_{\text{Imp}, i}}{\lambda_{\text{Imp}, i}} \right)^2 \right].
\]

(10)

The results of the fit with the basic forward model (\(d^{(1)}\)) have indicated residuals which are due to experimental aspects described in Sec. IV A 2 b.

2. Extensions (\(d^{(2)}\) . . . \(d^{(7)}\))

a. Hardware based extensions of the forward model.

The data (\(d\)) on the CCD-chip are given in pixels and must be mapped onto a wavelength scale. For this purpose, natural lines from a neon spectral lamp are detected on one channel. For the proof-of-principle a linear pixel to wavelength mapping using two neon lines (\(\lambda_{\text{Ne}1} = 650.65 \text{ nm}, \lambda_{\text{Ne}2} = 659.87 \text{ nm}\)) was sufficient.\(^{17}\) To improve the accuracy of the wavelength mapping a quadratic dispersion relation was applied by using three natural neon lines (\(\lambda_{\text{Ne}1} = 650.65 \text{ nm}, \lambda_{\text{Ne}2} = 653.26 \text{ nm}, \lambda_{\text{Ne}3} = 659.87 \text{ nm}\)). Additionally, a channel dependent shift of the wavelength scale (\(\Delta \lambda_{\text{c}}\)) due to imperfections of the optics was added.\(^{18}\) It turns out that the grating of the spectrometer is sensitive to small changes in the ambient temperature. To allow for the resulting variation in dispersion, the dispersion relation is calculated with each plasma discharge. Thus, the modelled data are calculated by \(d^{(2)} = d^{(1)}(\lambda(\text{pixel}), \Delta \lambda_{\text{c}}(\text{channel}))\).

Since a frame transfer CCD-camera is used, smearing on the detector is generated during each frame transfer (vertical shift). This adds onto all spectra on the CCD-chip and needs to be accounted for in the model: \(d^{(3)} = d^{(2)} + d_{\text{CT}}\). The smearing between the channels on the CCD-chip is estimated by combining the calibration data obtained from a covered channel, \(d_{\text{C}_{\text{in}}}\), with a channel dependent binning factor, \(C_{\text{bin}}\), (to gain higher signals several rows are binned to one channel) and the model parameter for smearing. \(C_{\text{sm}}\): \(d_{\text{CT}} = d_{\text{C}_{\text{in}}} \cdot C_{\text{bin}} \cdot C_{\text{sm}}\).

b. Experimentally based extensions of the forward model.

Additionally, aspects taken into account are non-ideal beam grids which result in focus astigmatism of the beam and the FIDA signal.

The first aspect is deviations of beam direction and width between the three energy components in the applied MSE geometry. This can also be observed in beam-into-gas calibration experiments.\(^{19}\) Here, no magnetic field is applied during neutral beam injection into a gas. The observed spectrum consists of three Doppler shifted beam emission lines. Each of these lines belongs to a beam energy component and does not overlap with the others, since they are purely Gaussian shaped due to the absence of magnetic field. Thus, separate widths (\(\sigma_{\text{MSE}}\) in Eq. (7) becomes \(\sigma_{\text{MSE}, i}\), with \(i = \{1, 2, 3\}\)) and deviations in positions (\(dE_{\text{in}}, \text{with}\ i = \{1, 2, 3\}\)) can be calculated and incorporated into the forward model for each beam energy component: \(d^{(4)} = d^{(3)}(\sigma_{\text{in}})\) and \(d^{(5)} = d^{(4)}(dE_{\text{in}})\), respectively.

The broad FIDA signal overlaps with the whole MSE spectrum but is of low intensity.\(^{20}\) In order to avoid the high modelling effort required for the small contribution of the FIDA signal, this component is approximated by two overlapping Gaussians of low heights at distinctly different wavelengths and with a large width of \(\approx 1.5 \text{ nm}\) (channel dependent): \(d^{(6)} = d^{(5)} + d_{\text{FIDA}}\).

\[
d_{\text{FIDA}} = \sum_{i=1}^{2} A_{\text{FIDA}, i} \exp \left[ -\frac{1}{2} \left( \frac{\lambda - \lambda_{\text{FIDA}, i}}{\sigma_{\text{FIDA}, i}} \right)^2 \right].
\]

(11)

c. Constraints of the forward model. The CX emission line is partly covered with only the right- and left-hand side wings available to be fitted, and the MSE spectrum is overlaid with the flat FIDA and smearing signals. This can cause the fit to reach a local minimum with physically unrealistic spectra. Thus, strict boundary conditions are imposed for several parameters, e.g., \(T_{\text{CX}}\) and \(\lambda_{\text{CX}}\): \(d^{(7)} = d^{(6)}(\text{boundary conditions})\).

B. Model and fit validation

To assess the enhancements of the different parts of the extended forward model (hardware and experimentally based parts as well as constraints) and to compare these approaches with the mathematical one (multi-Gaussian based simplex model), the reduced goodness-of-fit was calculated for each part. In these calculations the influence of the increase of the number of free parameters, which are outlined in the right table of Fig. 4 is included. In the left plot of Fig. 4, the calculated \(\chi^2_N\) for each model are presented for three different channels. It can be seen that even the basic physically motivated approach leads to much higher accuracy than the multi-Gaussian model. The use of a more accurate quadratic dispersion relation instead of the linear one and consideration of the smearing between the channels in the forward model reduces \(\chi^2_N\) significantly. Further reductions of \(\chi^2_N\) are achieved when taking into account experiment based aspects as deviations of beam width and beam energy for each beam energy component and the FIDA signal. The fit of the modelled data with constraints, especially for fit parameters describing the CX components prevent the fit from running into local minima and thus leads to better fit results with higher accuracy.

The reason for the improvements by fitting to a forward model is the matching of the spectral features of interest to the physical model making use of the entire spectrum. Fitting of individual lines (even with constraints) is much more sensitive to experimental errors and (local) background subtraction.

As outlined previously, the MSE spectrum is described by a model with only two quantities (\(E_{\text{f}}, T_{\text{b}}\)) and a set of fit parameters. However, the forward model uses all fit parameters leading to a complicated multivariate likelihood function underlying the fit procedure.

For an error discussion we assess a low dimensional projection of the likelihood function, cf. Figs. 5 and 6. The
FIG. 4. (a) Comparison between the multi-Gaussian based simplex model, the forward model and the extended forward model, latter regarded for its components separately ($d^1$, $d^2 = d^1 +$ quadratic dispersion, $d^3 = d^2 +$ cross-talk, $d^4 = d^3 +$ separated width for each beam energy component, $d^5 = d^4 +$ deviation term for variation in beam direction, $d^6 = d^5 +$ deviation term for variation in beam direction, $d^7 = d^6 +$ FIDA, $d^8 = d^7 +$ boundary conditions) for three different channels. (b) At the right table the number of free parameters, $f_i$, considered in $\chi^2$ ($N = \#data points - f_i$) is outlined for each model.

FIG. 5. Results of the error analysis of the MSE spectrum (grey-color coded is the log-likelihood $l$-normed with respect to maximum ($l/l_{max}$)$^{-1}$).
projections are chosen to assess both the uncertainties of the fit parameter and their respective correlations. For visualisation, both the log-likelihood and the chosen model parameters are normalized with respect to the best fit value. In this representation, the best-fit value is for all plots at \((x = 1, y = 1)\) with a minimum \(\chi^2\) of 1.95.

The error bars in Figs. 5 and 6 show the 2σ error of the fit parameters. The ellipses in the two-dimensional projection show the 2.3 · \(\sigma\) (containing 68.3% of the data) and 11.8 · \(\sigma\) (containing 99.73% of the data) iso-contours, respectively. The tilt of the contours reflects the linear correlation of the model parameters displayed. The chosen parameters are the measurable quantities \(E_L\) and \(T_p\) and some model parameters relevant to experimental set-up (\(A_0\), \(\alpha\), and \(C_{sm}\)). It is shown that significant correlation shown-up in the error analysis of \(E_L\) with \(\alpha\) and of \(T_p\) with \(A_0\) and \(C_{sm}\).

The Lorentz field was found to be independent of \(A_0\) and \(C_{sm}\). This is indicated by the circular (\(A_0\)) and non-inclined elliptical (\(C_{sm}\)) distribution of the data. However, there is a weakly negative linear correlation between \(E_L\) and the observation angle, shown in the middle left hand plot. The negative sign follows the direction of the ellipse’s main axis. To fulfil the same confidence interval, an increase in one parameter enforces a decrease in the other and vice versa. The different scaling of the x- and y-axes need to be considered in the middle left plot. \(T_p\) is linearly and negatively correlated to the beam amplitude and smearing factor, but not connected to the observation angle. This agrees with the expectation for \(T_p\) which depends on the intensity as well as the beam amplitude, while the observation angle depends on the wavelength. The opposite is true for the Lorentz field, which is calculated from the Stark-splitting and in consequence is dependent on the wavelength but not on the intensity. Hence, it is not surprising that \(E_L\) and \(T_p\) are not correlated to each other, as reflected in Fig. 6. Finally, the estimated precision of \(E_L\) is \(±0.02\%\) at the outermost channel and rises to \(±0.09\%\) at the innermost channel with respect to the plasma core. The increase in uncertainty can be explained by the decrease in the signal-to-noise ratio due to beam-stopping.

V. MEASUREMENT OF THE MAGNETIC FIELD

As an example of the application of the MSE diagnostic, Fig. 7 displays results from sMSE measurements for the ASDEX Upgrade discharge #26323 (\(B_{tor} = 2.48\) T, \(I_p = 0.8\) MA). The timings of the heating power by neutral beam injection (NBI) and electron heating (ECRH) are displayed in the upper panel of Fig. 7 showing a stepwise increase in heating power up to 10.8 MW. The heating is provided by four 2.5 MW NBI sources: the more tangentially off-axis heating power of NBI6, the more radially on-axis heating power of NBI8 and NBI5 are added to beam heating of NBI3 used for the sMSE diagnostic. To prevent Wolfram density peaking in the plasma center ECRH heating power of 0.8 MW is applied. The discharge is fuelled by gas puffing and the electron density is \(n_{co} = 6.3 \ldots 6.7 \times 10^{19}\) m\(^3\). The second row in Fig. 7 shows the central electron temperature and density, estimated by integrated data analysis (IDA\(^{21}\)), along with ion temperature measurements from charge exchange recombination spectroscopy.\(^{22}\)

The third row in Fig. 7 reflects stepwise increasing heating power in the apparent intensity ratio, \(T_p^{app}\). This effect increases towards the plasma core and innermost observation channel (at \(R = 1.74\) m and \(z = 0.09\) m). The Lorentz field also changes when the maximum heating power is applied as seen in the fourth panel of Fig. 7. This effect is less obvious than the change in \(T_p^{app}\) but is still visible. In order to examine the temporal evolution of \(T_p^{app}\) and \(E_L\), the bottom row of Fig. 7 shows more details along with exponential fits to the temporal evolution. The Lorentz field increases on time scales of \(\delta(t) \approx 80\) ms when switching off NBI5 (T2), whereas the polarization fraction relaxes with \(\delta(t) \approx 120\) ms. This figure shows typical NBI slowing down times of about 150 ms. These findings indicate the temporal resolution of the sMSE for equilibrium reconstruction.

In the phase of greatest heating power, an increase in the central pressure can be observed from kinetic measurements, as well as a decrease after switching off NBI5 (second row of Fig. 7). The kinetic pressure change (taken from kinetic measurements) is compared with the (dia)magnetic pressure change derived from the sMSE measurement (\(\Delta p \approx -\Delta E_L \cdot E_L \cdot B_0 \cdot \mu_0\)) for two time points in Fig. 8. These time points indicate different heating scenario transitions which are marked in Fig. 7 with \(T_{ON}\) before and after switching NBI5 on (\(t = 2.91\) s \(→ t = 3.42\) s) and \(T_{OFF}\) before and after switching NBI5 off (\(t = 3.42\) s \(→ t = 4.14\) s). The radial dependency shows good agreement of these two independent methods for estimating pressure change. Both techniques show small pressure variations in the outermost channel and an increase in the pressure change towards the plasma core. The pressure changes for both heating scenario transitions are about the same for all channels (\(\Delta p(T_{ON}) \approx -\Delta p(T_{OFF})\)).

The error bars indicate the 1σ interval of the error considering uncertainties in \(B_{tor}\) and \(E_L\). The latter is estimated by averaging over a period of almost constant plasma.
FIG. 7. sMSE results: Time traces for the measured Lorentz field $E_L$ and the apparent polarization ratio $T_{\text{pns}}^\text{sMSE}$ (third and fourth rows) for two ASDEX Upgrade shots (sMSE observed on NBI3) and a detailed view of the quantities of interest with exponential fit for the temporal evolution (lower panel). Time traces of the port-through power of applied heating sources is shown in the first row, the second row shows the central electron temperature ($T_e$) and density ($n_e$) (IDA21) along with ion temperature measurements ($T_i$) from charge exchange recombination spectroscopy.

FIG. 8. Pressure change for heating scenario transition of $T_{\text{ON}}$: before and after NBI5 on (black curves) and $T_{\text{OFF}}$: before and after switching NBI5 off (grey curves) obtained from the fitted Lorentz field (bold lines with circle-like symbol) and from kinetic measurements (dashed lines) for different radial positions.
1.70 1.75 1.80 1.85 1.90 1.95 ...

FIG. 9. Radial profile of fitted Lorentz field during high power phase (black curve) and after high power phase (grey curve). The radial shift of the magnetic axis for these time points is shown by the vertical lines in the same color.

Conditions (about 30 ms after the end of the slowing down phase) during and after high heating power, \( t = 3.18 \ldots 3.60 \text{ s} \) (\( \approx \) high power) and \( t = 3.78 \ldots 4.20 \text{ s} \) (\( \approx \) low power).

Possible reasons for the magnetic pressure variation can be extracted from the radial profiles of \( E_L \) for the two discharge periods (Fig. 9) in conjunction with results of equilibrium calculations. The equilibrium was calculated solving the Grad-Shafranov equation with CLISTE constrained by inductive magnetic measurements and a pressure profile which has been calculated with data from the kinetic measurement and from IDA. The fraction of the fast ion pressure is neglected. Moreover, the \( q \)-profile was calculated with the determination of the \( q = 1 \) surface of the sawtooth inversion radius.

There is almost no change in the Lorentz field in the outermost channel (at \( R = 1.99 \text{ m} \)), cf. Fig. 9. In contrast, the channels \( R = \{1.90 \text{ m}, 1.86 \text{ m}, 1.77 \text{ m}\} \) reflect a diamagnetic behaviour of the plasma, i.e., in the higher power phase the Lorentz field and thus \( B_{tor} \) is lowered reflecting the increasing plasma diamagnetism even the central channel shows a small decrease in the Lorentz field. The diamagnetic behaviour can be explained by the Shafranov shift shown by the vertical lines in Figs. 9 and 11 in the high power phase. The magnetic axis shifts with decreasing NBI power from about \( R \approx 1.729 \text{ m} \) to about \( R \approx 1.715 \text{ m} \) towards the high field side. The resulting shift of the pressure profile leads to the variation in the Lorentz field which is calculated with an uncertainty of \( \pm 0.12\% \).

In Fig. 10, absolute values of the measured Lorentz fields from the Stark-splitting are compared with values derived from magnetic field reconstruction of the CLISTE code and the neutral beam geometry.

The difference in the outcome of both methods is less than 1.5% in absolute values. In addition, the similar shapes of the time traces of the Lorentz fields reflect a good agreement of less than 1% difference in relative values.

A comparison of \( T_{\nu \nu} \) with the pitch angle in MSE geometry, \( \gamma_{Cl} \), in Fig. 11 shows \( T_{\nu \nu} \) to be an experimental indication for the direction of \( E_L \) and \( \vec{B} \) in the MSE geometry. The profiles of \( T_{\nu \nu} \) (upper subplot) and \( \gamma_{Cl} \) (middle subplot) are increasing with increasing beam power (Fig. 11). This effect gets stronger towards the core and indicates a change in the poloidal field.

In Fig. 12 the variation of the quantisation axis \( (E_L) \) from \( E_{L1} \) to \( E_{L2} \) is observed with the LOS. Thus, the detected \( \pi_{1,2} \) and \( \sigma_{1,2} \) polarized emission lines and the observation angle on \( E_L, \theta \) vary for both cases. With Eq. (5) follows a change in \( T_{\nu \nu} \). Considering the fact that \( |\vec{B}_{pol}| \) is about 10% of \( |\vec{B}_{tor}| \) at the edge and much less in the plasma center it follows that \( T_{\nu \nu} \) captures changes mainly in \( |\vec{B}_{pol}| \).

The magnetic variation during the two time periods can also be observed in the profiles of the safety factor \( q \). These are calculated by the equilibrium solver and show a much stiffer shear of the magnetic configuration for the time after NBI was switched off, cf. lower subplot of Fig. 11.

Systematic influence of polarizing optical elements on MSE polarimetry measurements are reported. In our studies, consistently with these measurements, a discharge and spatially dependent systematic bias with respect to results from equilibrium solver, \( \Delta \gamma_{\text{bear}} \), has also been found. Therefore, a characterization of the polarizing properties of the optical elements (Fig. 1), such as the retardance of the polarimeter is essential. For this purpose in-vessel calibration measurements with a polarized light source were performed.
without plasma operation. For the specific case discussed here \((t = 2.92 \, \text{s}, \, R = 1.91 \, \text{m}, \text{and} \, z = 0.09 \, \text{m})\) the bias was decreased from \(\Delta \gamma_{\text{bias}} = 17.17^\circ\) to \(\Delta \gamma_{\text{cal}} = -2.78^\circ\) with a precision of \(\delta \gamma_0 = \pm 3.68^\circ\).

A second systematic effect is the non-statistical distribution of the Stark sub-levels. Including this effect decreases the bias to \(\Delta \gamma_{\text{non-stat}} = 0.18^\circ\) and \(\delta \gamma_{\text{non-stat}} = \pm 3.65^\circ\). Other cases show deviations (bias) up to \(\Delta \gamma_{\text{non-stat}} = 1.8^\circ\) with respect to CLISTE results. On the one hand, within the error bars we find a consistent agreement between the equilibrium solver and our independent measurements. Thus, qualitative changes in the direction of the magnetic field can be detected, but the required absolute accuracy for the current \(j\)-profile reconstruction could not be achieved.

VI. CONCLUSIONS

The implementation of a spectral Motional Stark diagnostics on ASDEX Upgrade has been reported. Fit functions based on forward modelling of the measurements have been employed to analyze the data. The best fitting forward model includes the Stark-splitting of three components of the neutral beam injection, background signals including fast ion \(D_e\) emission\(^{25}\) and non-statistical distributions of excited atoms.\(^{12}\) Moreover, technical influences such as a nonlinear dispersion, cross-talk of CCD pixels, deviation of beam direction and width between the three beam energy components have been shown to affect the results significantly. The forward model as introduced in this paper was as globally statistically consistent with the data as to result in a normalized chi-square of \(\chi^2/N = 1.9\). For this forward model all effects have been either physically justified or proven by additional laboratory studies. This approach is in contrast to (mathematical based) heuristic parametric models, such as a multi-Gaussian model and showed much better fitting results.

Since the sMSE employs almost the same viewing geometry as the MSE polarimetry, the findings of the forward model give indications for improvements in the polarimetry such as considerations of the influence of polarizing elements in the optical path.

The most prominent quantities of interest of the measurement, the Lorentz field and the polarization factor given by \(\pi\)- and \(\sigma\)-polarized line intensities, have been determined quantitatively. A detailed error discussion indicates the statistical uncertainties (precision) and correlations of statistical errors in the quantities of interest and further model parameters, e.g., the beam intensity or the cross-talk in the CCD-chip. The statistical errors for the Lorentz field and the polarization factor are almost independent. The determination of \(E_i\) is not significantly correlated with the intensity ratio. This independence of polarization states is an advantage compared to MSE polarimetry especially in devices with high densities and temperatures, such as ITER (International Thermonuclear Experimental Reactor). In ITER plasmas, plasma facing mirrors are expected to be coated which can change the polarization state of the emitted light significantly.\(^{26-28}\) Figures for the accuracy have been determined by systematic analyses of a well diagnosed plasma discharge at different heating scenarios, from comparisons with equilibrium modelling (CLISTE\(^2\)) and by comparison with independent measurements of the plasma pressure.

The Lorentz field reflecting the total magnetic field has been determined reliably to a channel dependent accuracy of \(<1.5\%\) and a channel dependent precision of \(<0.1\%.\) These figures allow determination of the diamagnetic effect. The polarization factor indicates the direction of the magnetic field.
and can be determined with high precisions. However, the accuracy suffers from systematic polarization effects such as from the plasma facing window. It is noted that these issues are circumvented in polarimetry by additional calibration measurements which could also be used for sMSE. Nonetheless, small changes in the polarization factor in different heating scenarios have been clearly revealed.

It is concluded that the sMSE diagnostics has proven to provide reliably magnetic field strengths and precisely changes of its direction. Indications on the impact of polarization effects suggest permanent monitoring of the polarization properties of observation optics even for polarimetry.

In order to generalize the applicability of the forward model, non-Gaussian and asymmetric line shapes of the MSE spectrum should be included. It has been shown in Ref. 29 that this effect can significantly influence the MSE spectrum.

For further physics modelling, the sMSE data will be included in equilibrium modelling in a next step.

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5.3 Manuscript III

On the influence of Zeeman effect on magnetic equilibrium reconstruction using Motional Stark Effect diagnostic
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On the influence of Zeeman Effect on magnetic equilibrium reconstruction using Motional Stark Effect diagnostic

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Abstract.
The MSE diagnostic is a well established technique to infer the local internal magnetic field in fusion plasmas, but in order to reveal the small (dia)magnetic effects in the magnetic field measurements the necessary accuracy is quite demanding. With regard to this requirement an existing forward model which describes the MSE data is extended by the Zeeman effect, the fine-structure and relativistic effects in the interpretation of the Motional Stark effect (MSE) spectra and studies the influence of the Zeeman effect onto the spectrum of the Balmer-α beam emission for different experimental conditions at tokamak ASDEX Upgrade. This extension has an effect of about 2.2% in the derived magnetic field strength and of about 0.7° in the pitch angle compared to the pure MSE atomic model under ASDEX Upgrade conditions. The calculated magnetic field data are found in good agreement with the results from an equilibrium reconstruction solver.

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Zeeman-Stark Effect

1. Introduction

The accurate measurements of the local magnetic field is a quite demanding task in fusion plasmas and the Motional Stark effect (MSE) diagnostic represents probably the most sensitive and suitable instrument to deliver the necessary information. In general, the MSE concept relies on the observation of the Balmer-α transition \((n = 2 \rightarrow 3)\) emitted from injected high energetic \((10..100 \text{ keV/u})\) deuterium or hydrogen particles with velocity \(\vec{v}\) excited by collisions with plasma ions and electrons. The plasma is confined by background magnetic field on the order of 1..5 T. The observed emission is split into the nine observable Stark components by the Lorentz electric field \(\vec{E}_L\), \(\vec{E}_L = \vec{v} \times \vec{B}\), acting on atoms in their co-moving frame of reference, where \(\vec{B}\) is a local magnetic field vector. The resulting \(\sigma (\Delta m_l = 0)\) and \(\pi (\Delta m_l = \pm 1)\) spectral lines of the Stark multiplet are polarized parallel and perpendicular to the electric field direction, respectively. Here, \(\Delta m_l\) is the variation of magnetic orbital momentum. Therefore, the polarization of the observed lines is sensitive as to the orientation of the vector \(\vec{E}_L\) but also to the direction of the vector of magnetic field \(\vec{B}\) in the plasma. Employing polarization measurements from the central, unshifted \(\sigma_0\) line it is possible to reconstruct the pitch angle of magnetic field by the MSE polarimetry system [1, 2, 3]. In spectral MSE measurements the line splitting, \(\Delta \lambda\), depends on \(|\vec{E}_L|\) and therefore it allows us to measure \(|\vec{B}|\) [4, 5, 6]. The MSE diagnostic is routinely used as a tool to improve the equilibrium reconstruction [7, 8, 9, 10, 11]. However, the desired high precision for magnetic field measurement could be not achieved due to a number of inaccuracies in earlier analysis such as the treatment for the population densities of excited magnetic sub-levels [12, 13, 14, 15]. In addition to that, the Zeeman effect was often neglected in the beam emission analysis with regard to its smallness compared to the Stark effect [13]. In this paper, the effect of magnetic and electric fields on the Balmer-α emission is revisited. The atomic physics of combined Zeeman-Stark effect [16, 17, 18, 19] is adapted for the application in MSE measurements and the Zeeman effect and fine-structure is discussed in view of the spectral MSE observations. The model is prepared for even more refinements which can be done in future, e. g. by including contributions of radial electric fields. The recently developed MSE forward model [20] is extended and takes the Zeeman-Stark effect and the spin-orbit coupling into account in order to describe the measured MSE spectra. Finally, the results of the measurements are compared with results from equilibrium solver (CLISTE) [21] and transport code (TRANSP) [22] calculations for ASDEX Upgrade experimental conditions.

2. Atomic model of the Zeeman-Stark multiplet

Atomic model of hydrogen atom in the presence of electromagnetic field represents a topic that is still far from being closed, specially in studies of high Rydberg states or in the case of strong fields [23], though the experimental data for the simplest configurations are understood now. So for instance, it is an established fact that the Zeeman effect, or more precisely the Paschen-Back effect, dominates the fine-structure splitting of the Balmer-α line emission at the plasma edge of fusion devices [24]. In case of Maxwellian distribution function of atoms every magnetic component of the spectral line is described using a Doppler profile taking into account the different source of excited atoms [24, 25]. In the case of MSE measurements the emissions
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takes place predominantly in the static crossed electric and magnetic fields, being a
subject of studies for high Rydberg states [23, 26]. The influence of the fields onto the
emission pattern of Balmer-α line in fusion plasmas was shown in [16, 18] and for MSE
observations in [27]. The energy displacement of the levels caused by the magnetic
field depends on the mutual orientation between the vectors \( \vec{E} \) and \( \vec{B} \) [23]. In the
first order perturbation theory the linear and quadratic dependence on the strength
of magnetic field appears in the energy expression if vectors are orthogonal to each
other [28]:

\[
E^{\pm}(n, k) \approx \pm \Omega + k \sqrt{\left( \frac{3}{2} n F \right)^2 + \Omega^2},
\]

(1)

Here, \( E^{\pm}(n, k) \) is the energy of levels with \( n = n_1 + n_2 + |m_l| + 1 \), where \( n \) is the
principal quantum number, \( m_l \) is the orbital magnetic number, \( k = n_1 - n_2 \) is the
electric quantum number and integers \( n_1 \) and \( n_2 \) are parabolic quantum numbers,
with \( 0 \leq n_1 < n \) and \( 0 \leq n_2 < n \). The parameter \( \Omega = 1/2 \cdot \vec{B}/B_0 \) is the
magnetic field strength \( (B_0 = 2.35 \cdot 10^5 \text{ T}) \) and \( F = E_L/E_0 \) is the electric field
strength \( (E_0 = 5.142 \cdot 10^{11} \text{ V/m}) \). The expression 1 is valid only if \( F, \Omega \gg \delta \),
where \( \delta \) is the fine structure splitting. Two effects caused by magnetic field are observed from
expression 1. First, the magnetic field increases efficiently the electric field strength
of the pure parabolic states. Second, the linear term removes their double degeneracy
due to the interaction of spin magnetic moment with the magnetic field. In case of
MSE observation the ratio between the electric and magnetic field remains constant
(\( \vec{E}_L = \vec{v} \times \vec{B} \)) and as in majority of cases \( F > \Omega \), the expression 1 reduces to:

\[
E^{\pm}(n, k) \approx \frac{3}{2} n F \left( k \pm \frac{k^2}{2 n^2} \right),
\]

(2)

with

\[
\zeta_n = \frac{\Omega}{3/2 n F} \quad \text{(3)}
\]

\[
= \frac{2}{3n} \cdot \frac{v_0}{v \sin \omega},
\]

(4)

considering only one term of expansion in \( \Omega/(3/2 n F) \). Here, \( v_0 = 2.188 \cdot 10^6 \text{ m/s} \)
is atomic unit of velocity, \( v = \sqrt{2E/m} \) is the velocity of beam atom in m/s, and \( \omega \)
is the angle between vectors \( \vec{B} \) and \( \vec{v} \). Parameter \( \zeta_n \) characterizes the impact of
magnetic field on the displacement of energy levels for MSE observations. Similar to
the contribution of magnetic field in the final expression for energy one could also
estimate the relative contribution of the fine-structure splitting relative to the Stark
effect [29]. In this case:

\[
\zeta_{n}^{fs} \approx \frac{\alpha^2}{n^3} \cdot \frac{2}{3n F},
\]

(5)

where \( \alpha = 1/137 \) is the fine-structure constant. Substituting the energy of beam
atoms on the order of 10 keV/a.m.u. and magnetic field of 2 T, which corresponds
to the condition of third energy components at ASDEX Upgrade, one obtains for the
levels of \( n = 3 \) the values \( \zeta_3 = 0.35 \) and \( \zeta_3^{fs} = 0.08 \) and for levels of \( n = 2 \) the
values \( \zeta_2 = 0.52 \) and \( \zeta_2^{fs} = 0.4 \). Obviously, the magnetic field and the fine-structure
splitting could be not neglected in the description of the MSE spectra at this low
‡ The atomic units are used in this section.
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atom energies. The impact of these effects is different for MSE observation. Thus, if the fine-structure splitting of $n = 2$ shifts the transition of the D$_\alpha$ line as a whole, the linear term of magnetic field in 2 does not have any effect on the Zeeman-Stark pattern as the same terms in energy expression are presented in both upper and lower levels for observed lines 1. In contrast to this the magnetic field changes the purity of the new states, so that the $\sigma$- and $\pi$- transitions contain different polarization fractions. The second effect plays more important role in the MSE spectra analysis as depending on the observation geometry the shift caused by fine-structure alone could be negligibly small relative to the Doppler shift of the beam atoms. The general considerations shown above must be observed in the atomic data, e. g. energy levels and line intensities measured in crossed fields.

The calculation of atomic data in crossed static electric and magnetic fields was performed in frame of the perturbation theory of the basis of the field-free wavefunctions in the reference frame as shown in Fig. 1. In this coordinate system the Lorentz field $\vec{E}_L = \vec{v} \times \vec{B}$ is taken to be parallel to the $z$-axis and the vector of magnetic field ($\vec{B} = (B, 0, 0)$) is aligned along the $x$-axis. The vector of the velocity $\vec{v}$ is depicted to be in the $x$-$y$ plane ($\vec{v} = (0, -v, 0)$). The direction of observation is shown by the vector $\vec{s}$ with the polar angle $\phi$ and the azimuthal angle $\theta$. The plane normal to the vector $\vec{s}$ defines the direction of the orthogonal polarization vectors $\vec{e}_1$ and $\vec{e}_2$ so that $\vec{e}_1 \cdot \vec{e}_2 = 0$. In addition, we choose the vector $\vec{e}_2$ to be parallel to the xy plane. The

![Figure 1. Frame of reference and vectors orientation used in the present calculation: $\vec{B}$ is the vector of magnetic field, $\vec{E}_L$ is the vector of induced Lorentz field, $\vec{v}_\perp$ is the vector of atom velocity, $\vec{s}$ denotes the direction of observation, $\vec{e}_1$, $\vec{e}_2$ are polarization vectors, $\phi$ and $\theta$ are the angles determining the observation orientation. The electric field induces linear polarized emission in the direction parallel to $\vec{E}_L$ ($\pi_{\vec{E}_L}$), circular polarized emission perpendicular to $\vec{E}_L$ ($\sigma_{\vec{E}_L}$); the magnetic field induces linear polarized emission in the direction parallel to $\vec{B}$ ($\pi_{\vec{B}}$) and circular polarized emission perpendicular to $\vec{B}$ ($\sigma_{\vec{B}}$).](image)

energies of the new eigenstates in crossed fields, as shown in Fig. 1, were obtained by diagonalizing the Hamiltonian of the atom. The latter includes the relativistic effects, fine-structure splitting, and operators of interaction of atom with electric and magnetic fields. We note that the Lamb-shift being on the order of 0.0353 cm$^{-1}$ for $n = 2$ levels compared to 0.365 cm$^{-1}$ of fine-structure separation was not included in our calculations. The details of calculations in crossed fields could be found elsewhere [18]. In all cases the results reproduced well the cases of pure Zeeman and Stark effects. In Fig. 2 we show the example of calculation of $n = 2$ energy levels in crossed
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Figure 2. Energy levels of \( n = 2 \) in crossed electric and magnetic fields. The energy, strength of magnetic and electric field are shown in units of the field-free splitting \( \delta = 0.365 \text{cm}^{-1} \) between \( j = 3/2 \) and \( j = 1/2 \) levels. The zero in the ordinate corresponds to the non-relativistic energy of the \( n = 2 \) levels, so that \( E(j = 3/2)/\delta = -1/4 \) and \( E(j = 1/2)/\delta = -5/4 \). The ratio \( \Omega/F = 0.79 \) (beam energy is 10 keV/a.m.u. and magnetic field is 2 T) is kept constant in the calculation. a) case of weak electric and magnetic fields: \( F, \Omega \approx \delta \). Thin dashed lines show the energy of the levels for Stark effect only \( (\Omega = 0) \), solid lines correspond to calculation of Zeeman-Stark effect. b) case of strong fields: \( F, \Omega \gg \delta \), where different colors corresponds to states with different \( m_l \) numbers (parabolic states). Dashed-point lines for the outermost components show the Zeeman-Stark effect calculations neglecting the spin of the atoms, for other lines the notation is the same as in a).
interaction of magnetic field with orbital momentum, e.g., quadratic term in equation 2, increases the displacement in energy of the new states as shown by dashed point lines for two outermost components. Finally, the interaction of magnetic field with spin momentum split every of the levels (dashed-point lines) into two \( \pm \Omega \) components relatively to the Stark states (solid lines). In Fig. 3 we show the results of calculations for the experimental conditions relevant in fusion plasmas for intensity of \( H_\alpha \) line. First, we consider the case without magnetic field as shown in Fig. 3.a. In the pure MSE case with \( \vec{E}_L \) pointing into the z-direction the pattern consists of fifteen lines with an equidistant line splitting, nine of which are, in practice, detectable. The individual transition lines are perpendicular polarized (\( \sigma \)) or parallel polarized (\( \pi \)) to \( \vec{E}_L \) components. For each polarization state the sum over all lines, including weak ones, is conserved so that \( \sum_{ij} I^\pi = 1/2 \sum_{ij} I^\sigma \). The relative intensities calculated with this approach (dashed lines) agree with calculations [31], [30], p. 277 and also

Figure 3. Calculation of \( D_\alpha \) multiplet for the beam energy of 10 keV/a.m.u and magnetic field of 2 T. Shift of the polarization components are shown in the units of \( 3/2 F \) (a.u.), blue lines denote the \( \sigma \) components, red lines denote the \( \pi \) components. Fine-structure field-free calculations are shown as black thin lines to indicate the scale of the splitting. a) Stark effect calculations: solid lines show the results of calculation taking the fine structure into account; dashed lines with dots are Stark effect calculation in strong field, e.g., fine-structure and relativistic effects are neglected. b) Solid lines show the results of Zeeman-Stark effect calculations and dashed lines - results of calculations in strong fields as in (a). c) Fraction of \( \sigma \) components at \( \pi \) Stark lines and \( \pi \) components at \( \sigma \) Stark lines due to the Zeeman effect. Here, the intensity of all lines in vicinity of corresponding transition was summed up.
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field-free (thin solid lines) line strengths, $\sum_{\pi}^{\pi} I_{\pi} = 36.907$ a.u. [32]. By including the fine-structure in the calculations one shifts the energy of the whole multiplet and splits the components with final states $m_I = \pm 1$ according to the results of Fig. 2. In Fig. 3.b one observes the impact of magnetic field on the multiplet. By neglecting the spin of the atom one observes the same picture as in case of pure MSE but the line positions are shifted due to the quadratic term in Eq. 2. This shift is less than the corresponding displacement induced by fine-structure as discussed before (dashed lines). By taking spin of the atom and fine-structure into account one observes the splitting of the components due to the linear term of interaction (solid lines). The following consequence for the MSE diagnostic can be observed. One detects the redistribution of the polarization pattern, e.g., the pure Stark $\pi$ transitions obtain the small fraction of the $\sigma$ contribution and on the other hand the pure Stark $\sigma$ transitions obtain certain fraction of $\pi$ components. In all cases the sum over all $\sigma$ and $\pi$ components remains constant, though the different polarizations appear at the same positions compared to the Stark effect. In order to exemplify this effect we show the fraction of $\sigma$ components at Stark-$\pi$ lines and $\pi$ fraction at Stark-$\sigma$ lines in Fig. 3.c. One observes the mixing on the order of $1 - 3\%$ due to Zeeman effect. The strongest mixing of polarization is observed at $\pi_4$ and $\pi_2$ lines. The fraction is shown at position of Stark lines, though the emission takes place at slightly different positions as shown in (b). Thus, the aim of this paper is to analyze the impact of the mixing of polarization components and of the line shift to the experimental data and moreover, to determine their effect on the pitch angle and on the magnetic field, respectively.

3. Comparison with experimental data

We now discuss the differences of the pure Motional Stark effect and Zeeman-Stark effect (ZMSE) model for parameters relevant to the experimental results. In case of MSE model we consider the simplest picture of strong field, neglecting the spin of the atom. For the given experimental conditions Fig. 4 (a.) shows the modelled Doppler-shifted emission pattern for both calculations, MSE and ZMSE, normalized to their maximum value. For the magnetic field of $|B| = 2.2$ T and ASDEX Upgrade relevant beam energies $E_0 = 29.8$ keV/a.m.u., $E_{1/2} = 14.9$ keV/a.m.u., $E_{1/3} = 9.95$ keV/a.m.u. one observes the pattern represented by the blue, red and green curves. The MSE results are plotted using solid lines and the ZMSE results are represented by dashed lines. The ZMSE pattern is plotted in yellow and only slightly deviates from the MSE pattern (black). To reveal the spectral differences between both models the residuum $I_{ZMSE} - I_{MSE}$ is plotted in Fig. 4 (b.). The obtained difference between both models is up to 4% with respect to the maximum intensity. The main cause for the big difference in the measured intensity is the shift of the line position. It is noted that the observed difference is strongly related to the chosen geometry setting ($\vec{E}_L$, $\vec{B}$ and $\vec{s}$, cf. Fig. 1). For observation of the emission along $\vec{E}_L$ ($\theta = \pi$) all polarization directions perpendicular to $\vec{E}_L$ will be observed, ($\pi_B$, $\sigma_B$ and $\sigma_{E_L}$). At line-of-sight parallel to $\vec{B}$ ($\theta = \pi$, $\phi = \pi$), all multiplet components which are perpendicularly polarized to $\vec{B}$ are observable ($\sigma_B$, $\sigma_{E_L}$ and $\pi_{E_L}$).

In order to discuss the geometry dependence, Fig. 5 (a.) and (b.) show the difference between MSE and ZMSE calculated spectra in dependence on the orientation of observation. Here the observation angles $\phi$ and $\theta$ are varied from $\phi = [0, \pi]$ and...
Figure 4. (a.) Doppler shifted beam profile for both MSE (black curve) and combined ZMSE (yellow curve). The MSE (solid lines) and ZMSE (dashed lines) for the individual ASDEX Upgrade beam energies are plotted in blue (full energy component, $E_0 = 29.8 \text{ keV/a.m.u.}$), red (half energy component, $E_{1/2} = 14.9 \text{ keV/a.m.u.}$) and green (third energy component, $E_{1/3} = 9.95 \text{ keV/amu}$). A typical ASDEX Upgrade magnetic field of $|\vec{B}| = 2.3 \text{T}$ was applied. In (b.) the residuum between both, ZMSE- and MSE-spectrum is plotted.

\[ \theta = [0, \pi/2]. \] The calculation was done for a beam energy of $E_0 = 30 \text{ keV/a.m.u.}$, the magnetic fields was set to $2.3 \text{T}$.

For almost all observation angles the Zeeman effect leads to an increase of the observed sum of $\pm \pi_2$, $\pm \pi_3$, $\pm \pi_4$ lines, and, at the same time, a decrease of the observed $\pm \sigma_1$, $\sigma_0$ lines. The black box in Fig. 5 indicates the region of ASDEX Upgrade geometry. Here, the difference in the spectra results in about 0.35% at the position of Stark $\pi$ component and $-0.25\%$ at the position of Stark $\sigma$ component. The changes of the line intensities have impact on the observed line ratio $T_p = \sum(I_{\pi})/\sum(I_{\sigma})$, where the sum is extended over the $\pm \pi_2$, $\pm \pi_3$, $\pm \pi_4$ or $\pm \sigma_1$, $\sigma_0$ lines. This parameter is of crucial importance for the derivation of the pitch angle $\gamma$. The question is, how this affects the pitch angle, $\gamma$. We introduce the pitch angle, which measures the direction of the Lorentz field projected in the MSE geometry:

\[ \gamma = \arctan \frac{E_{Lz}}{E_{Lx}}. \] (6)

The orientation of $\vec{E}_L$ is determined by the observation angle $\theta$ and the direction of the beam. The angle $\theta$ is a function of the observed line ratio

\[ \theta = \arccos \sqrt{\frac{1 - T_p}{1 + T_p}}. \] (7)

Fig. 6 shows pitch angles calculated from the spectral pure MSE (black) and spectral ZMSE data for three ASDEX Upgrade beam energies (blue, red, green). The dashed
black lines represent a typical \((T_P, \gamma)\)-relation as found in the measurements applying the forwards model of MSE measurements and are used for the discussion below. For the three beam energies the same \(T_P\) value leads to the corrected pitch angles indicated by symbols. The effect on the pitch angle measurement is \(\Delta \gamma = \{0.31^\circ, 0.52^\circ, 0.74^\circ\}\) is quite significant compared to the required accuracy for fusion devices which is in the range of 0.1°...0.5° [4]. It is concluded that pitch angle reconstructions suffer systematically from a neglection of the Zeeman effect and its correction is almost in the order of about 1\%. 

As shown in Sec. 2 the Zeeman effect and the fine-structure cause a shift of the multiplet and a change in the line splitting. For ASDEX Upgrade relevant conditions the multiplet is shifted by about 5\% for 30 keV/a.m.u. to 11\% for 30 keV/a.m.u. beam energies with respect to the \(\sigma_0\)-Stark line. The line splitting changes in the range of 1\% (30 keV/a.m.u) to 2\% (30 keV/a.m.u.). In Fig. 7 the change in \(|\vec{B}|\) due to the difference of the line splitting between pure MSE case and ZMSE case is shown for varying splitting and ASDEX Upgrade beam energies. The splitting is the mean value taken from most intensive lines (\(-4\pi...+4\pi\)). The scattered symbols denote experimental data taken from a magnetic field ramp-down discharge (#26322), the inclined lines represent the fit referred to the experimental data. The color code corresponds to the beam energies. For a magnetic field of about 2.3 T a difference of 1.6\% \((E_0)\)...2.5\% \((E_{1/3})\) can be seen. This is a significant effect and needs to be considered for the calculation of the absolute value of \(B\).

The aforementioned formulation of the ZMSE case with the spin-orbit coupling and relativistic effects is now included into the forward model and the measured spectral MSE data, \(\vec{d}\), at ASDEX Upgrade are fitted using the forward model [20]. The forward model
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Figure 6. Pitch angle variation due to changes in line ratio for three different ASDEX Upgrade beam energies. The dashed black lines mark a typical \((T_p, \gamma)\)-relation when applying the spectral pure MSE. The symbols mark the \(\gamma\) value for the same \(T_p\) value but calculated with the spectral ZMSE, including fine-structure and relativistic effects. The colors indicate the certain ASDEX Upgrade beam energy.

Figure 7. Magnetic field variation as a function of the line splitting at the radial position \(R = 1.78\) m. The crosses represent pure MSE case (along the solid lines) and ZMSE with fine-structure and relativistic effects (along the dashed-dotted lines) calculated splitting corresponding to a magnetic field ramp performed during ASDEX Upgrade discharge 26322. The lines along the experimental data represent a fit to these data. The horizontal black line indicate a magnetic field calculated with CLISTE corresponding to a MSE splitting value (vertical lines). The dashed horizontal lines represent the magnetic field values corresponding to the ZMSE Model evaluated splitting value (vertical lines). The data are represented color-coded for the three beam energies full (black), half (blue) and third (red).

describing the measured data consists of a constant background signal \((d_{BG})\), carbon impurity lines \((d_{Imp})\), active charge exchange \((d_{CX})\), a FIDA signal \((d_{FIDA})\) and the ZMSE pattern \((d_{ZMSE})\). Moreover, the cross-talk on the CCD-chip during readout process \((d_{CT})\) is included into the forward model:

\[
d(F_{EL,B,L-S}) = d_{ZMSE} + d_{CX} + d_{BG} + d_{Imp} + d_{FIDA} + d_{CT} \quad (8)
\]
The extension of the forward model in [20] is to include the Zeeman effect and the effect of the spin-orbit coupling and relativistic effects in the description of the Balmer-α emission. This was done by extending the pure MSE model with correction factors for the wavelength splitting and for the intensity relation of the σ and π-polarized Stark lines.

The model of the pure MSE spectrum considers all 15 (σ and π) Stark components with a spectral profile function constructed by a Gaussian. To consider the different energies, three MSE spectra are modelled using the amplitude, $C_{bi}$, the Doppler shifted position of the central σ0 line, the lines position, $\lambda_{ELi,\sigma}$ and the line ratio $T_P$:

$$\vec{d}_{\text{MSE}} = \sum_{i=1}^{3} C_{bi} \left( T_P \sum_\pi A_\pi \exp \left[-\frac{1}{2} \left( \frac{\lambda - \lambda_{ELi,\pi}}{\sigma_w} \right)^2 \right] + \sum_\sigma A_\sigma \exp \left[-\frac{1}{2} \left( \frac{\lambda - \lambda_{ELi,\sigma}}{\sigma_w} \right)^2 \right] \right).$$  \hspace{1cm} (9)

The fitting parameters are $C_{bi}$, $E_L$, $T_P$, the line shift and the width, $\sigma_w$. Thus, for the modelling of one MSE multiplet five free parameters were used. The Einstein coefficients $A_{\pi,\sigma}$ for the π and σ lines of the Stark spectrum are taken from [31]. The width is mainly affected by the beam width and the instrument function. For the wavelength mapping a quadratic dispersion relation was determined by three natural neon lines ($\lambda_{\text{Ne1}} = 650.65 \text{ nm}$, $\lambda_{\text{Ne2}} = 653.29 \text{ nm}$, $\lambda_{\text{Ne3}} = 659.90 \text{ nm}$). Non-statistical distribution of sub-levels are considered by a density, magnetic field and beam energy dependent parameter, $c_{ns}$, that was calculated by a collisional-radiative model [12] and used as a correction factor for $T_P$

$$T_{P}^{ns} = c_{ns} \cdot T_{P}. \hspace{1cm} (10)$$

It need to be noted that factor $c_{ns}$ is in the range of $0.8 \pm 0.04$, which implies an angle correction of about $\Delta \gamma_{ns} \approx 3^\circ$.

In order to take into account changes in the line ratio and the line mixing effect in the ZMSE case shown in Fig. 3 and Fig. 5, a correction for the line ratio $T_P$ has to be done analogue to the statistical plasma correction in Eq. 10. Thus, the corrected line ratio is

$$T_{P}^{ns,\text{ZMSE}} = c_{T_P} \cdot T_{P}^{ns}. \hspace{1cm} (11)$$

To consider the line splitting of the ZMSE pattern in the forward model the calculated splitting difference between MSE- and ZMSE-model is implemented line dependent in the forward model

$$\lambda_{(EL,B)}_{i,\sigma} = \lambda_{ELi,\sigma} + \Delta \lambda_{(EL,B)}_{i,\sigma}. \hspace{1cm} (12)$$

Thus the full description of the ZMSE pattern in the forward model is:

$$\vec{d}_{\text{ZMSE}} = \sum_{i=1}^{3} C_{bi} \left( T_P \sum_\pi A_\pi \exp \left[-\frac{1}{2} \left( \frac{\lambda - (\lambda_{ELi,\pi} + \Delta \lambda_{(EL,B)}_{i,\pi})}{\sigma_w} \right)^2 \right] + \sum_\sigma A_\sigma \exp \left[-\frac{1}{2} \left( \frac{\lambda - (\lambda_{ELi,\sigma} + \Delta \lambda_{(EL,B)}_{i,\sigma})}{\sigma_w} \right)^2 \right] \right). \hspace{1cm} (13)$$
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3.1. Reference discharge

In order to validate the forward model, a reference discharge has been conducted on ASDEX Upgrade. The discharge parameters were chosen to reflect conditions which have been analysed with the CLISTE equilibrium code [33, 21]. Fig. 8 shows the time evolution of the discharge indicating a stationary plasma current of \( I_p = 0.8 \) MA (a.) and a stationary heating of \( P = 5.8 \) MW (b.) but a decrease in the absolute value of the toroidal magnetic field from \( |B_{\text{tor}}| = 2.6 \) T to \( |B_{\text{tor}}| = 2.4 \) T (a.). \( B_{\text{tor}} \) has been decreased by lowering the toroidal field coil current.

Fig. 9 shows the temporal evolution of the Lorentz field \( \vec{E}_L = \vec{v} \times \vec{B} \) from both an independent analysis of CLISTE (blue) and from the fitted data of the previously discussed MSE forward model (green) and ZMSE forward model (red). The panels represent different locations as indicated by their respective \( R \) and \( z \) values. The CLISTE data are directly derived from \( \vec{E}^{\text{CL}}_L = \vec{v}_\perp \times \vec{B}^{\text{CL}} \), where \( \vec{v}_\perp \) is taken from calibration measurements of the Beam and MSE geometry and \( \vec{B}^{\text{CL}} \) is a result of the solution of the Grad-Shafranov-Equation [34] in CLISTE. The forward modelled Lorentz fields are calculated with the Schwartzschild-Epstein equation [31]. The CLISTE calculations were constrained by magnetic measurements, \( q \) and the total pressure \( (p_{\text{tot}} = p_{\text{kin}} + p_{\text{FI}}) \). Since sawtooth activity has been observed, the safety factor was set \( q = 1 \) at the axis. In fact this is not exact but setting \( q = 1 \) at the inversion radius \( (\rho_0 \approx 0.23) \) lead to almost the same results. The kinetic contribution of the total pressure, \( p_{\text{kin}} = k_B (n_e T_e + n_i T_i) \), was obtained from kinetic measurements and integrated data analysis (IDA) [35]. The fast ion pressure contribution, \( p_{\text{FI}} \), was calculated with the transport code TRANSP [22].

The linear ramp down phase of about 6 % between \( t = 3.8 \) s and \( t = 6.2 \) s was assumed to follow the linear decrease of \( B_{\text{tor}} \) and fitted by a linear model. The precision for each channel was estimated from the sum of the squared residuals. The resulting 2\( \sigma \) error intervals are represented by the shaded regions and are about the same order for CLISTE and forward model data. However, in contrast to the CLISTE data the precision of the forward model data was found to be channel dependent. With
σ = 0.3 % the error is the lowest at the outermost channel and rises towards the
plasma core with a maximum value of σ = 0.6 % for the innermost channel. This can
be explained by the beam attenuation which leads to a decreasing signal-to-noise level
towards the plasma.

The results show a small radius dependent difference between ZMSE and MSE model
up to 2.5 % and a good agreement for the temporal variation between both methods.
In all cases the derived Lorentz and magnetic field for MSE case are higher as in case
of ZMSE model which is in agreement with results of Sec. 2 (Fig. 3). Indeed, the
magnetic field causes additional splitting of the components so that weaker Lorentz
electric field is now required to describe the measured spectra. The MSE data are
found even in slightly better agreement with CLISTE calculations as ZMSE results.
The difference between the MSE and ZMSE data reduces towards the plasma core,
which could be explained by the stronger attenuation of the third energy component.
But this has to be analyzed in near future. The total error in the variation of the
Lorentz field is ∆E_L/E_L0 ≈ 0.5 %. The reasons for the channel dependent error could be:

(i) Imperfections in the optics components in the MSE set-up: e.g. by non-optimal
adjustment of the detection components which consists of a spectrometer, an
objectives and a CCD-chip. The MSE diagnostic is described in detail in [20]
(ii) Use of a improper profile function for the MSE lines: in the present work a
gaussian profile was applied. However, this is not exact. Dux has shown in [36]
that the MSE profile is asymmetric due to the variation of the magnetic field
along the line-of-sight when it is crossing a beam with a certain width. The effect
is the strongest in the innermost channel.

![Figure 9](Image)

**Figure 9.** Discharge #26322: Time traces of the Lorentz field calculated with the
CLISTE equilibrium code with run# 2364 (blue), with the ZMSE forward model
(red) and with the MSE forward model (green): For all methods the fit functions
(straight lines) and the related rmse confidence intervals (shadowed regions) are
given.

It can be concluded that local variations in the magnetic fields of less 0.5 % can be
detected. Moreover, the spectral ZMSE diagnostic can be used for the measurement of
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absolute values of the local magnetics with a high accuracy of about 1% or even better. The measured values have a high precision between 0.3% and 0.6%. To improve the consistency with CLISTE results in the measurement of the absolute values the difference in the bias has to be minimized. This could be done by applying asymmetric MSE profile functions and by increasing the accuracy. However, the findings show that the application of the ZMSE forward model is a suitable tool to confirm and, moreover, to improve equilibrium reconstructions.

4. Summary and Outlook

In this paper the influence of the Zeeman effect was analysed for the measurements of MSE spectra at the ASDEX-Upgrade tokamak. The contribution of these effects to the Balmer-α beam emission spectrum has been investigated systematically for different geometry, beam energy and magnetic field strength. It was found that under typical ASDEX Upgrade conditions the line splitting is affected by the ZMSE in the range of 1% for 30 keV/a.m.u. to 2% for 10 keV/a.m.u. deuterium beam energies. The changes in the observed line ratio $\sum_i I_i / \sum_j I_j$, with $i = \{\pm 2, \pm 3, \pm 4\}$ and $j = \{\pm 1, 0\}$ due the Zeeman effect are up to 2% (10 keV/a.m.u.). The discrepancies for the energy dependent line splitting and line ratio were included into the new ZMSE forward model as correction parameter. The resulting changes in the absolute value of the magnetic field are about 1.6% (30 keV/a.m.u.) to 2.5% (10 keV/a.m.u.) which is in the range of the para- and diamagnetism. The calculated pitch angles differ about 0.7° from the analysis based on the atomic models of pure Stark effect. This is significantly higher than the required accuracy for fusion devices which is in the range of 0.1° to 0.5°. From these finding it can be concluded that the accurate modelling of the Zeeman-Stark effect is required to fulfill the needed accuracy for the determination of the magnetic field strength. We note that the present analysis was performed in the first-order perturbation theory only. Also the results of non-statistical model in pure parabolic Stark states and impact of the Zeeman effect on the line ratios was not taken self-consistently into account. We are going to improve this model in the near future.

The extended forward model was validated with an ASDEX Upgrade discharge. The applied linear decrease of the toroidal magnetic field of about 6% could be reconstructed by the forward ZMSE model. The calculated Lorentz fields show a channel dependent offset of $\Delta E_{L,0} \approx 0\%$ to 2.5% and a difference in the inclination of about $\Delta(\delta E_{L})/E_{L,0} \approx 0.5\%$ compared to Lorentz fields calculated with the equilibrium solver CLISTE. We could show that the ZMSE forward model leads to slightly lower Lorentz fields compared to the MSE forward model. This is consistent with results from the atomic physics calculations, which showed that the line splitting is increased by the Zeeman effect and the fine structure. The high accuracy in both, the absolute value and the time development demonstrates the spectral MSE diagnostic with the forward model of ZMSE to be a suitable tool for accurate equilibrium reconstruction. The error estimated from the statistical noise is slightly lower then the error of the CLISTE data for the outer channels but increases towards the inner channels due to beam attenuation. Further improvements could be the reduction of the noise by improved hardware settings, e.g. using not the optical path of the polarimeter set-up. Furthermore the uncertainty of the data have shown the need of a full statistical description of the forward model, for example by a bayesian approach. Moreover, the forward model
Zeeman-Stark Effect can be refined by considering additional electric field components, e.g. radial electric field.

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5.4 Manuscript IV

Combined Zeeman and Motional Stark Effect measurements of local magnetic effects on ASDEX Upgrade
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Combined Zeeman and Motional Stark Effect measurements of local magnetic effects on ASDEX Upgrade

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Abstract.
In view of accuracy requirements to resolve fast ion induced effects on the magnetic equilibrium, a comprehensive physics forward model is applied on the Balmer–α line. For the first time, the Zeeman Effect and the Motional Stark Effect (MSE) are considered in the model to analyze the spectral MSE data of a high-β discharge with a stepwise increasing Neutral Beam Injection (NBI) heating power. The calculated magnetic field data as well as the revealed (dia)magnetic effects are consistent with the results from an equilibrium reconstruction solver. The related fast ion pressure variations derived from the spectral Zeeman and Motional Stark Effect (ZMSE) forward model data agree well within their error range with the fast ion pressure changes calculated by a transport code.

Keywords: Motional Stark effect, combined Zeeman and Motional Stark effect, beam emission spectroscopy, magnetic field measurements, magnetically confined plasmas, plasma diamagnetism, fast ions in tokamaks
1. Introduction

The magnetic configuration of a magnetically confined plasma is strongly related to the local plasma pressure and the current profile. The magneto-hydrodynamic force balance \( \nabla p = \vec{j} \times \vec{B} \) describes a condition for stationary magnetic equilibria. Changes in the fast ion population can cause diamagnetic effects which decrease the toroidal magnetic field by about 1\% \cite{1,2}, and these small effects, though difficult to detect, are of great importance for the local state of the plasma. Even more difficult is the measurement of changes to the local poloidal magnetic field (due to current profile reconfiguration) \cite{3,4,1}. Consequently, the detection of these small variations requires highly sophisticated techniques, including corresponding qualified data analysis.

In this paper spectral Motional Stark effect measurements of the internal local magnetic field \cite{5,6} are performed. The concept relies on the observation of the Balmer-\(\alpha\) transition \((n = 3 \rightarrow 2)\) from highly energetic injected deuterium particles which are excited by collisions with ions and electrons. The beam particles have a velocity \(\vec{v}_b\) with respect to the background magnetic field \(\vec{B}\). For practical purposes, the emission is split into 9 observable Stark components by the electric Lorentz field, \(\vec{E}_L = \vec{v}_b \times \vec{B}\), acting on atoms in their moving frame of reference. The resulting \(\pi\) \((\Delta m_l = 0)\) and \(\sigma\) \((\Delta m_l = \pm 1)\) lines of the Stark pattern are polarized parallel and perpendicular, respectively, to the local Lorentz field. Therefore, the polarization of the Stark lines is sensitive to the orientation of \(\vec{E}_L\). From the line splitting, \(\Delta \lambda\), the Lorentz field and thus \(|\vec{B}|\) can be deduced \cite{7,8,5}.

In a former work \cite{9} we found that the Zeeman effect and the fine-structure affect the line splitting by about 1\% and the intensity relation by about 3\% for a mid-sized Tokamak. Combined with the MSE it forms the so-called combined Zeeman-Stark Effect pattern \cite{10,11,12,13}.

In this paper the combined Zeeman Motional Stark Effect is exploited by a forward model to measure variations in the fast ion pressure profiles in a high-\(\beta\) discharge scenario. The results are consistent with results obtained from equilibrium solver \((CLISTE)\) \cite{14} and transport code \((TRANS\)P) \cite{15} calculations. Moreover, magnetic pitch angle measurements were performed and compared to \textit{CLISTE} results.

2. Fast ion effects in NBI heated high-\(\beta\) discharge

2.1. Discharge overview

In order to assess the potential sensitivity of spectral MSE measurements to fast-ion effects, a discharge with stepwise increasing heating power up to 10.8 MW was conducted within this work. Purpose of the experiment was to examine the effect on the plasma equilibrium. Fig. 1 shows relevant time traces of discharge\# 26323 on ASDEX Upgrade. Fig. 1 (a.) indicates the applied heating: Electron cyclotron heating (ECRH) was applied in order to prevent tungsten accumulation in the plasma center \cite{16,17,18}. Neutral beam injection (NBI) heating with deuterium beams was provided by four 2.5 MW NBI sources for \(t > 1.2\) s. The more tangentially off-axis deposited heating power of the injected NBI6, the more radially on-axis heating power of NBI8 and NBI5 are added to beam heating of NBI3 used for the sMSE diagnostic. Details about the geometry of the applied beams can be seen in Fig. 2 which shows the toroidal (a.) and poloidal view (b.) of ASDEX Upgrade. Fig. 1 (b.) indicates the total toroidal plasma current with \(I_p = 0.8\) MA during the flat-
top phase ($t > 0.8 \, s$) and the external toroidal magnetic field of $B_\phi = -2.48 \, T$. Fig. 1 (c.) and (d.) show the temperature and density: the black lines represent the central electron temperature ($T_e$) and central electron density ($n_e$) determined by the integrated data analysis diagnostic (IDA). The red lines indicate the central ion temperature measurements ($T_i$) from charge exchange recombination spectroscopy and the central ion density ($n_i$) resulting from $n_e$ and $Z_{eff}$ [19, 20]. The latter has a value of about $Z_{eff} \approx 1.5$. The periodic oscillations in the kinetic signals, especially in the ion and electron temperature time traces reflect the occurrence of sawtooth activity in the plasma. The main aspects of the discharge are the stepwise increase and decrease of the NBI heating power at time points indicated by the vertical dotted lines.

2.2. Fast ion pressure variation deduced from the forward modelled Lorentz field variation

In this section the variation of both, total and fast ion pressure are derived from the Lorentz field as an application of the spectral combined Zeeman-Stark effect diagnostic. The results are compared to results of the equilibrium solver CLISTE and the transport code TRANSP.
In Fig. 3 (b.) the time traces of the central kinetic pressure, derived from the given experimental data $p_{\text{kin}} = k_B \cdot (n_e T_e + n_i T_i)$, the central fast ion pressure, $p_{FI}$, gained from the transport code TRANSP and the central magneto-hydrodynamic pressure, $p_{\text{mhd}} = p_{\text{kin}} + p_{FI}$, are presented. Furthermore, the stored fast ion and magneto-hydrodynamic energies, calculated with TRANSP are given in (b.). The corresponding time evolutions of the Lorentz fields and pitch angles calculated with the MSE forward model and the forward model of the combined Zeeman and Motional Stark effect are shown in (c.) and (d.) for a central channel.

The NBI sources differ in the direction of injection (Fig. 2 and [21]), which is of importance when discussing equilibrium results, the NBI heating sources mainly generate fast ions in the direction of heating. NBI3, NBI5 and NBI8 point more perpendicular and only NBI6 more parallel to the magnetic field. Thus there is a higher production of fast ions with perpendicular velocity, which results in an anisotropic fast ion pressure. The TRANSP results confirm this and show a relation for the fast ion pressure of $p_{\text{FI}, \perp}/p_{\text{FI}, \parallel} \approx 1.3$. However, the applied equilibrium solver CLISTE does not take into account pressure anisotropy. Thus the fast ion pressure is assumed to be isotropic for the forthcoming analysis.

The time traces of the central total pressure and central total energy reflect the heating pattern: additional NBI heating leads to a rise and reduced NBI heating leads to a decrease of these quantities. The diamagnetic decrease in the magnetic field due to the rise in the total pressure can be observed in the decrease of the modelled Lorentz field in (c.). This behaviour is mainly related to changes in the toroidal magnetic field whereas variations in the pitch angle, shown in (d.), are mainly related to changes in the poloidal field. According to the findings in [9] the Zeeman Effect does not significantly change the shape of the Lorentz field and the pitch angle but contributes as an offset in these magnetic quantities.

As depicted in Fig. 3 (b.) additional NBI heating not only increases the thermal plasma pressure but also increases the production of high energetic particles (fast
ions), which gyrate around their guiding center and thus induce a magnetic field component almost anti-parallel to the toroidal magnetic field. The high contribution of the fast ion pressure in the total pressure of more than 30% indicates that the generated fast ions lead to detectable changes in the magnetic configuration and need to be considered in equilibrium reconstruction. This effect is reduced for lower total pressures.

In Fig. 4 the time evolution of the Lorentz field calculated with the improved forward model (a.) is compared to results of CLISTE (b.) for five different radial positions. On top of the figures the applied method is labelled. The CLISTE run was constrained by external magnetic measurements, the safety factor on the magnetic axis ($q_{ax} = 1$) and by the total pressure profile.

The time traces of CLISTE calculated signals show a significant response on the heating variation consistent to the findings for the forward modelled Lorentz fields in Fig. 3 (c.). The stepwise increase and decrease of the NBI heating power lead to a change in the measured Lorentz field followed by an exponential decay phase. As similar to findings discussed in the previous section, the ZMSE data show a lower noise level for the outer channels than CLISTE data. Towards the plasma core the noise level of the ZMSE data rises due to the beam attenuation. In order to calculate
the Lorentz field variation due to changes in the heating scenario the \textit{CLISTE} and forward model data were fitted with an exponential decay:

\[
E(t) = E_0 + \Delta E \left(1 - \exp \left(\frac{t_0 - t}{\tau_D}\right)\right),
\]
with the fit parameter \(E_0\) denoting the Lorentz field at the beginning of each heating phase, \(\Delta E\) denoting the amplitude of the change of the Lorentz field and \(\tau_D\) the decay time. The latter fit parameter is a measure for the confinement times in ASDEX Upgrade. The obtained values differ in a range of 20 ms ... 160 ms with a high uncertainty of about 50 ms due to the high noise and low time resolution in the data. However, these times agree in magnitude with the known slowing down times of fast ions and with the energy confinement time for the ASDEX Upgrade, which are about 60 ms. \(t_0\) and represents the onset-time of each heating scenario phase. All four parameter are dependent of the heating interval and of the position \((R, z)\). The shaded area indicates the 1\(\sigma\) interval of confidence of the fit. The channel dependent deviation of 0.45\% (Ch1) ... 1\% (Ch4) with a mean deviation of \(rms = 0.7\%\) indicates a good agreement between these models for this discharge. Both models show a similar response on the heating variation in the calculated Lorentz field.

From the related Lorentz field variation the total pressure variation can be deduced using the pressure balance equation in cylindrical approximation

\[
\frac{dp}{dr} + \frac{B_\theta}{\mu_0 r} \cdot \frac{d(r B_\phi)}{dr} = j_\phi B_\phi, \tag{2}
\]
with the poloidal current density

\[
j_\phi = -\frac{1}{\mu_0} \frac{dB_\phi}{dr}. \tag{3}
\]
the magnetic permeability $\mu_0$ and the minor radius $r$. For the diamagnetic limit, where the pressure gradient is the dominating part Eq. 2 can be reduced to

$$\frac{d}{dr} \left(p + \frac{B_0^2}{2\mu_0}\right) = 0$$

(4)

The measured Lorentz field is mainly related to the toroidal magnetic field. Thus modifications in the toroidal magnetic field can be approximated by Lorentz field variations and the pressure balance equation for the diamagnetic limit can be written as:

$$\Delta p \approx -\Delta E_{DL} \cdot \frac{B^2}{\mu_0}$$

(5)

In order to take into account only the diamagnetic effect ($\Delta E_{DL}$), Lorentz field changes due to the Shafranov shift ($\Delta E_{S}$) need to be subtracted from the measured total Lorentz field variation ($\Delta E_L$). The contribution of the Shafranov shift to the total field variation is calculated by the CLISTE equilibrium code and is approximately given by $\Delta E_L^S / E_L \leq 0.1\%$.

Fig. 5 (a.) and (b.) show the variations of the pressure for the most significant cases when NBI5 is switched on (a.) and off (b.). Results from different methods TRANS (mhd), kinetic measurements (kin), forward model (ZMSE) and CLISTE (CL) are compared with each other. Consistent with the findings in Fig. 3 (b.), additional heat load leads to a rise and reduced heating to a decrease of the total pressure and kinetic pressure. The effect of the heating is most significant in the plasma center, here $|\Delta p_{tot}| \approx 40 \text{kPa}$ and $|\Delta p_{kin}| \approx 23 \text{kPa}$ when NBI5 is switched on. Towards the the plasma edge the pressure variation vanishes. This indicates that the pressure profile gradient increases with additional NBI heating and vice versa.

The pressure profile gradient calculated from the forward model data shows the same behaviour. In fact, within the errors, the forward modelled data (black bold line) show a good agreement with the total pressure results from TRANS (black dashed line) for both, NBI5 on and NBI5 off, cases. Moreover, these results are consistent with the determination of the total pressure variation by CLISTE. It should be noted that the CLISTE calculations showed low sensitivity to the pressure profile it was constrained with, which indicates that CLISTE is operating at its limit of sensitivity. In the error the channel and time dependent uncertainties of $\Delta E_L$, $E_L$ and of $B$ are included.

With the knowledge of the kinetic pressure change the fast ion pressure variation can be calculated. The results (black line with symbols) are compared with the TRANS calculations (red dashed lines) in the panels (c.) and (d.) of Fig. 5 for the transitions NBI5 on and NBI5 off. Although there are discrepancies of about 1 to 5 kPa the profiles shape agrees with each other and the data fit within their 1σ confidence interval. It can be concluded that with the spectral ZMSE diagnostic small changes in the magnetic configuration and, moreover, total pressure and thus together with the kinetic pressure from kinetic measurements the fast ion pressure variations can be detected.

The measured magnetic effects and the related pressure profile variations can be expressed by the plasma $\beta$ which represents the performance of the plasma. Considering Eq. 5 the local $\beta$ is deduced from forward model data and CLISTE data:

$$\Delta \beta \approx -2 \frac{\Delta E_L}{E_L}$$

(6)
Figure 5. Comparison of pressure profile variations for the heating scenario transition NBI5 switched on (a.) and (c.) and NBI5 switched off (b.) and (d.): the upper panels show the total and kinetic pressure variation, the lower panels present fast ion pressure variation. Error bars from error propagation equation taking into account the $1\sigma$ uncertainty of $\Delta E_L$, $E_L$ and $B$.

In Fig. 6 the local $\beta$ variation calculated with the forward model and with CLISTE is shown for the cases NBI5 on (a.) and NBI5 off (b.). It can be seen that with increasing heating the plasma leads to an increase of the local and global plasma $\beta$ (a.). The effect is up to $\approx 1\%$ in the plasma center and vanishes towards the outer region. The local increase is consistent with the observed local diamagnetic effect due to the rise of the local total pressure. Switching off the heating source NB15 has the opposite effect. The decreased total plasma pressure and increased magnetic field leads to a lower plasma confinement (b.). The agreement between equilibrium reconstruction data and the forward model data demonstrates the potential of the spectral ZMSE diagnostic to detect both, the total pressure variations and the related diamagnetic effects.

2.3. Pitch angle

As shown in [22] the MSE forward model allows the evaluation of the pitch angle from the ratio of the $\sigma$ and $\pi$ lines from the MSE spectrum [21, 22]. This is still valid for the extended model. Two other methods determining the pitch angle independently are the MSE polarimetry, which applies the central $\sigma_0$ line from the MSE spectrum and the equilibrium reconstruction by solving the Grad-Shafranov equation. In Fig. 7 the time traces of the forward modelled pitch angles (b.) are compared with time traces calculated by the equilibrium solver CLISTE (a.) and MSE polarimetry (c.).
All three methods are able to detect variations in the pitch angles due to changes in the plasma heating. Increasing NBI heating leads to a rise of the pitch angles and vice versa, except for the outer channels. Consistently to the results from Sec. 2.2 the effect is most significant in the center. The statistical noise is indicated with a shaded area. The remarkable low noise-level of the forward modelled data is about 0.12° (Ch1) . . . 0.21° (Ch4) which is about 30% of the MSE polarimetry noise-level. Similar to the Lorentz field data the beam attenuation lead to a high uncertainty in the Forward modelled pitch angle for the central channel and are not useful for the later analysis. An offset correction was necessary to bring the data at the same level. The correction has been performed by a minimizing model, that minimizes the difference \( \epsilon_1 \) between CLISTE and Forward model and CLISTE and MSE polarimetry data: 
\[ \epsilon_1 = d_{CLISTE} - d_{ZMSE} \] 
\[ \epsilon_2 = d_{CLISTE} - d_{MSEP} \] 
The channel dependent bias is given for each channel in the gray boxes in Fig. 7. As can be seen for this discharge, the offsets of both MSE diagnostics differ channel dependent within the range of 0.4°.

The effects responsible for the offsets are not yet fully understood, but are matter of current investigation [5, 23]. First results indicate that besides such apparatus effects as coating on the vacuum window, or ageing of the photo elastic modulators of the polarimeter set-up, reflection in the plasma vessel is a likely cause. The offset of the spectral diagnostic is \( \gamma_0(ZMSE) = \gamma_0(ZMSE) - 0.5° \). Due to the occurrence of the offset in the pitch angle the full potential of the spectral MSE diagnostic, the self-consistent calculation of the magnetic field, could not could not be applied. However, besides the calculation of absolute values of \( E_L \), the diagnostic can be applied to measure variations in the pitch angle, e.g. due to changes in the heating scenario.

All three methods, the equilibrium code CLISTE, the forward model and the MSE polarimetry diagnostic, showed the most significant changes in \( \gamma \) when NBI source 5 was switched on and after it was switched off again, cf. Fig. 7. In Fig. 8 the profiles of the pitch angle variation, calculated with the three methods, are presented for both, the NBI5 on transition phase (a.) and the NBI5 off transition phase (b.). It can be seen that for the first case \( \gamma \) decreases at the outer channels but rises at the inner channels. This is vice versa, for the second case, where NBI5 is switched
Figure 7. $\gamma$-comparison between forward model (a.), CLISTE (b.) and MSE polarimetry (c.) results. Forward model and MSE polarimetry results are corrected by an channel dependent offset. The offsets of both MSE diagnostics are given in the gray boxes for each channel. The shadowed regions indicate the $1\sigma$ error band.

off. Although the observed changes are small ($-0.5^\circ \ldots 0.5^\circ$), all the independent methods produced similar results. These facts and the aforementioned low noise level show that the spectral MSE results are trustworthy and demonstrates that the spectral MSE diagnostic fulfills required accuracies for fusion devices of about $0.1^\circ \ldots 0.5^\circ$ [1].

Figure 8. Variation of $\gamma$ due to variation in NBI heating: (a.) NBI5 switched on and (b.) NBI5 switched off. Forward model data are compared to CLISTE and MSE results.

3. Summary and Outlook

By employing the combined Zeeman Motional Stark effect on the hydrogenic heating beams a high resolution technique for the detection of small effects in the local
magnetic configuration has been developed on ASDEX Upgrade. A ZMSE forward model, as described in [9], was applied to determine fast ion variations in a high-β discharge scenario with stepwise increasing and decreasing NBI heating power. The rise of the fast ion pressure with additional NBI heating power could be determined from their measured local diamagnetic effect observed in the Lorentz field. The changes of the fast ion pressure of ≈ 0 kPa at the plasma edge to 15 kPa at the plasma center are consistent with results from TRANSPI and CLISTE. The improved plasma confinement β also derived from the Lorentz field variation agrees with predictions from the CLISTE. A reduction in the heating power lead to a reduction of the diamagnetic effect in the plasma. The fast ion pressure as well as the local β were decreased.

Effects of the fast ions in the pitch angle could be seen in the time development of γ and were compared to equilibrium reconstruction results of CLISTE and to MSE polarimetry data. The channel dependent precision of about 0.12° . . . 0.21° is about 30 % of the precision of the data of the MSE polarimetry. The observed channel dependent deviation of around −1° . . . −1.5° between ZMSE and CLISTE data are consistent with the offset which was also observed by the ASDEX Upgrade MSE polarimetry diagnostic. Once the offset can be determined by a physical model, the full potential of the spectral ZMSE diagnostic, a self consistent reconstruction of the magnetic field, can be exploited. Good agreement between CLISTE and forward model data were found for the pitch angle variation for chosen discharge scenario transitions.

Further improvements could be expected by the reduction of the noise by improved hardware settings, e.g. using a less complex optical path by omitting the polarimeter set-up. Furthermore the uncertainty of the data have shown the need of a full statistical description of the forward model, for example by a bayesian approach. Moreover, the forward model can be refined by considering additional electric field components, e.g. radial electric field.

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5.5 Article V

Forward modelling of Motional Stark Effect spectra
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FORWARD MODELING OF MOTIONAL STARK EFFECT SPECTRA

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A forward model for motional Stark effect spectra is developed. The forward model includes beam attenuation and considers the detection of partially polarized light. The forward model is applied to simulate spectra for Wendelstein 7-X. A fit version is used to analyze data from experimental spectra from ASDEX Upgrade. The fit validates the model and enhances the confidence in the simulation. The structure and the content of the model are described to allow modular implementations to guide physics-based diagnostic designs.

KEYWORDS: motional Stark effect, diagnostic design, polarization spectroscopy

Note: Some figures in this paper are in color only in the electronic version.

I. INTRODUCTION

Quantitative simulations of experimental data expected for future fusion devices require synthetic diagnostics. The resulting virtual instruments can be effectively used to guide the design of diagnostics based on the underlying physics. For existing setups, virtual instruments can be used for the simulation of measurements, and thus, measuring capabilities can be assessed, e.g., by evaluating spurious effects. Moreover, diagnostic upgrades or routine maintenance of diagnostics can be assessed by performance checks using the virtual instruments. When using a suite of synthetic diagnostics for the same physical quantity, the approach may also contribute to data validation by providing the possibility of cross-checks.

Probabilistic models of measurements naturally lead to synthetic diagnostics. A mathematical model to describe the error statistics of a measurement is the “likelihood” of the data. The likelihood function of the data is one key ingredient for Bayesian data analysis. A central element of the likelihood is the forward model, which maps deterministically all quantities and measurement parameters of the data. This is what synthetic diagnostics are supposed to do.

The specific example discussed in this paper is part of a broader approach that investigates requirements for synthetic diagnostics for next-step, steady-state fusion devices where increasing challenges are posed to diagnostics. Examples of diagnostic applications are machine protection, plasma control, and physics evaluation, which rely on diagnostic capabilities. Such steady-state devices demand much higher reliability and robustness because of the extended pulse lengths of the devices. In addition, high and sustained neutron fluxes may lead to new phenomena including radiation-induced luminescence or radiation-induced conductivity. Effects of measurement accuracy, resolution, and lifetime need to be assessed to comply with requirements for quantitative physics evaluation of experiments.

As seen by the ITER requirements for data analysis, which envisage an integrated design for diagnostics, simulations are to keep up with the highest demands on the confidence in the diagnostics models. Fusion diagnostics design is assumed to be both as predictive and as reliable as possible. These kinds of requirements have been addressed in related analyses done for Wendelstein 7-X (W7-X) (Ref. 6). With the accomplishment of an integrated design concept, a suite of virtual instruments
is expected to be viable. This suite can be regarded as a flight-simulator tool that allows diagnosticians to assess whether diagnostic units comply with the required measuring capabilities. Since, in addition, the impact of foreseeable constraints, possible faults, or unwanted effects can be simulated, this approach is preferable to trial-and-error steps.

This paper discusses the specific implementation of a general forward model for future spectral motional Stark effect (MSE) measurements on W7-X. First spectrally resolved MSE measurements have been reported in Ref. 7. Forward simulations of an active polarimetry MSE diagnostics8 have been reported in Ref. 9. The simulation of a spectrally resolved MSE diagnostic is specifically treated in Refs. 9 through 16. MSE is planned to measure \( q \)-profiles to control the current profiles in ITER (Ref. 4).

Simulations for ITER MSE diagnostics are reported in Ref. 17. Partly overlapping, partly newly developed, the forward model presented here involves a collisional-radiative beam-plasma model (CRM) simulating the beam attenuation. Alignment effects18,19 and resulting deviations in emission line ratios are not considered in the forward model, but the model is structured to include respective model extensions. In a fit version of the code, a parameter involving corresponding deviations (polarization factor) is included. For the simulations, the effect of alignment has been assessed by a correction to the simple CRM. A full simulation of polarization properties employing the Müller-matrix formalism for partially polarized light is implemented. This submodel can be transferred to other spectroscopic measurements. The code is structured and set up to be integrated in Bayesian analyses. Thereby, device-dependent parameters are separated from the model making the code device independent, similar to the codes reported in Refs. 10 and 11. Moreover, simplifications of time-consuming submodules were implemented to allow diagnostic optimization studies20 at feasible computational times.

In order to validate the chosen version of the forward model against experimental data, MSE spectra from ASDEX Upgrade are analyzed using a fit version of the forward model. This approach is chosen to enhance the confidence in the forward model while its structural design tries to emphasize the possibility to transfer individual parts of the model to other applications.

II. MSE FORWARD MODEL

The forward function is a deterministic mathematical representation of a measurement (free from errors). Since the forward function enters the misfit distribution of the data, the forward function is also a key element for the probabilistic description of measurements. To simulate data for future devices, predictive simulations are used to simulate the measurements. Equilibrium calculations give the \( \vec{B} \) field; plasma profiles are used to determine the beam excitation. The result of the simulation of the measurement are the data \( \mathcal{D} \). Figure 1 is a flowchart that shows the elements entering the simulation of the spectral MSE measurements.

The flowchart also gives a rough guideline for the modularization of the forward model. Each element can be regarded as a subpart of the model. For the implementations of the submodels, the arrows indicate interfaces. The interface defines the required content for the neighboring subpart. Keeping a more refined submodel compliant with the interface definitions allows one to exchange the subparts rather than setting up a complete new forward model. The model can be used for simulation, analysis, and diagnostics design and optimization as discussed above.

II.A. From Physics Effects . . .

The MSE is the Stark splitting in the Lorentz field due to the electric field \( \vec{F} \) in the frame of a fast beam moving in a strong magnetic field. \( \vec{F} \) consists of a Lorentz field, the component of interest in MSE diagnostics, and the radial electric field. A convenient choice is the
spectroscopy of the linear Stark effect on hydrogen isotopes for fusion diagnostics; the quadratic Stark effect leads to more complicated effects.\cite{21} For Balmer-alpha spectroscopy with beam energies on the order of 50 keV/amu and magnetic fields of 2.5 T, the energy shift $\Delta E_n$ in the Stark multiplet of hydrogen isotopes can be well described by the Epstein-Schwarzchild formula\cite{22}:

$$\Delta E_n \text{ (eV)} = 7.94198 \times 10^{-7} \times |F| \text{ (V m}^{-1}\text{)} n(n_1 - n_2),$$ \hspace{1cm} (1)

where $n$ is the principal quantum number and $n_1$ and $n_2$ are parabolic quantum numbers. The influence of $B$ in the moving frame has been considered by, e.g., Refs. 12, 18, and 23. For this study, the combined Stark-Zeeman effect is not considered since it leads to only minor deviations from the linear Stark effect. The Stark multiplet consists of $\Delta m = 0 (\pi)$ and $\Delta m = \pm 1 (\sigma \pm)$ transitions, where $m$ is the magnetic projection quantum numbers. The $\sigma$ and $\pi$ components are linearly or elliptically polarized depending on the direction of observation with respect to the electric field vector $\vec{F}$.

To start with the forward model, the signal level measured $D$ by a pixel of a charge-coupled-device (CCD) chip is given by

$$D\text{(pixel)} = \varsigma\text{(pixel)} \times \Pi \times \int_{A_p} \int_{\tau} \frac{\Delta L\text{(pixel)}(\lambda)}{\Delta \text{pixel}} \cos \xi \, dA_p \, d\Omega.$$ \hspace{1cm} (2)

The dispersion relation of the spectrometer $\text{pixel} = \text{pixel}(\lambda)$ is determined by wavelength calibration measurements. The radiance $L$ is the radiant flux per unit projected area observed at the boundary of the plasma $dA_p$ and solid angle $d\Omega$. Later, the radiance is represented by a Stokes vector; $\vec{L}$ is used to represent the polarization states of partially polarized plasma emission.\cite{24} $\Delta L$ refers to the wavelength interval covered by a pixel with width $\Delta \text{pixel}$. $\xi$ is the angle enclosed by the line of sight and the normal vector of the surface $A_p$. Here the emission will be integrated along a line of sight through an optically thin plasma. The line integration will enter the radiance (see below), and the observed plasma surface $A_p$ is given by the entendue $T \approx A_p \Omega_m$ in the direction of the line of sight. Therefore, the angle $\xi$ does not need to be accounted for (cos $\xi = 1$). Practically, $T$ is limited by the observation optics.\cite{25} $A_p$ is the most limiting cross section in the observation optics, and $\Omega_m$ is the corresponding solid angle. The operator $\Pi$ accounts for polarization-dependent transmission of the observation optics. The factor $\varsigma$ is the sensitivity that contains the quantum efficiency (photoelectron per photon) and the amplification of the detector (counts per photoelectron).

The radiance $L$ represents the radiation like from an emitting surface. $\varsigma$ is related to the line integral of the local emission coefficient $\epsilon$ along the line of sight $L$; writing the emission coefficient as a Stokes vector accounts for the polarization of the light:

$$L = \int_{L} \epsilon(s) \, ds,$$ \hspace{1cm} (3)

where $s$ is a coordinate along the line of sight. Radiation transport is neglected since the plasma is optically thin at wavelengths of the MSE spectra. Provided the variations of the observed cross sections $A_p$ are small and the emission coefficient $\epsilon$ across the line of sight is small, Eq. (2) becomes

$$D(\text{pixel}) \Delta \text{pixel} = \varsigma(\text{pixel}) \times T \times \Pi \times \int_{L} \Delta \epsilon(s, \text{pixel}(\lambda)) \, ds.$$ \hspace{1cm} (4)

So far, the forward model contains only algebraic operations representing a model for the detection of partially polarized light. To link the signal with the beam excitation process, physical models for the emission coefficient being represented as a Stokes vector $\hat{\epsilon}$ need to be provided. A first, simplified model consists of the MSE spectrum, the charge-exchange emission from the beam. This simple model disregards beam particle diffusion (beam halo) or effects resulting from fast ions. The motivation of this simplification results from the data analysis described later: It gives a reasonable description of the data, and it avoids, e.g., modeling of fast ion charge-exchange processes requiring large computational effort. The simple MSE model reads

$$\hat{\epsilon}(s, \lambda) = \hat{\epsilon}_{\text{MSE}}(s, \lambda) + \hat{\epsilon}_{\text{CX}}(s, \lambda) + \text{const},$$ \hspace{1cm} (5)

where a constant background due to bremsstrahlung is accounted for.

The simple forward model for the emission coefficient $\Delta \epsilon(s, \text{pixel})$ can be readily extended to include further effects:

$$\hat{\epsilon}(s, \lambda) = \hat{\epsilon}_{\text{MSE}}(s, \lambda) + \hat{\epsilon}_{\text{halo}}(s, \lambda) + \hat{\epsilon}_{\text{FIDA}}(s, \lambda) + \ldots \hat{\epsilon}_{\text{edge}}(s, \lambda) + \hat{\epsilon}_{\text{Imp}}(s, \lambda) + \hat{\epsilon}_{\text{BS}}(s, \lambda).$$ \hspace{1cm} (6)

In this more complete model, the emission coefficient is assumed to consist of contributions from the MSE, charge exchange of the plasma with the beam also covering fast ions (FIDA) (this contribution includes $\epsilon_{\text{CX}}$), light from the beam halo (halo), emission from the cold plasma edge (edge), impurity lines (Imp), and bremsstrahlung background (BS). All components contribute to different polarization states $p$.

The comparison of Eq. (5) with Eq. (6) reflects how the concept of modular subparts can be assigned to physical submodels. In the case considered here,
submodels can be either exchanged (\(\varepsilon_{\text{CX}}\) by \(\varepsilon_{\text{FIDA}}\)) or included.

On the way to a specific implementation, we write down explicit dependencies of the individual components:

\[
\begin{align*}
\hat{\varepsilon}_{\text{MSE}}(s, \lambda) &= \sum_{\alpha} \left( \frac{hc}{4\pi\lambda} \bar{S}_{\alpha} \sum_{\sigma} n^{(\alpha, 3, \sigma)}_{\text{beam, } \alpha}(s) \right. \\
&\quad \times A_{\alpha} \mathcal{P}_{\beta}(s, \lambda, \lambda_{\alpha, \sigma}) \ldots \\
&\quad + \left. \frac{hc}{4\pi\lambda} \bar{S}_{\alpha} \sum_{\sigma} n^{(\alpha, 3, \sigma^{-})}_{\text{beam, } \alpha}(s) \right) \\
&\quad \times A_{\alpha} \mathcal{P}_{\beta}(s, \lambda, \lambda_{\alpha, \sigma^{-}, \alpha}) \right),
\end{align*}
\]

(7)

\[
\begin{align*}
\hat{\varepsilon}_{\text{CX}}(s, \lambda) &= n^{(\alpha=3)}_{\text{plasma}}(s) \frac{hc}{4\pi\lambda} \bar{S}_{\text{CX}} A_{\alpha=3} \mathcal{P}_{\beta}(\lambda) \ ,
\end{align*}
\]

(8)

\[
\begin{align*}
\hat{\varepsilon}_{\text{halo}}(s, \lambda) &= n^{(\alpha=3)}_{\text{halo}}(s) \frac{hc}{4\pi\lambda} \bar{S}_{\text{halo}} A_{\alpha=3} \mathcal{P}_{\beta}(\lambda) \ ,
\end{align*}
\]

(9)

\[
\begin{align*}
\hat{\varepsilon}_{\text{edge}}(s, \lambda) &= n^{(\alpha=3)}_{\text{edge}}(s) \frac{hc}{4\pi\lambda} \bar{S}_{\text{edge}} A_{\alpha=3} \mathcal{P}_{\beta}(\lambda) \ ,
\end{align*}
\]

(10)

\[
\begin{align*}
\hat{\varepsilon}_{\text{FIDA}}(s, \lambda) &= n^{(\alpha=3)}_{\text{FIDA}}(s) \frac{hc}{4\pi\lambda} \bar{S}_{\text{FIDA}} A_{\alpha=3} \mathcal{P}_{\beta}(\lambda) \ ,
\end{align*}
\]

(11)

\[
\begin{align*}
\hat{\varepsilon}_{\text{Imp}}(s, \lambda) &= \sum_{\text{Imp}} n_{\text{Imp}}(s) \frac{hc}{4\pi\lambda} \bar{S}_{\text{Imp}} A_{\text{Imp}} \mathcal{P}_{\beta}(\lambda, \lambda_{\text{Imp}}) \ ,
\end{align*}
\]

(12)

and

\[
\begin{align*}
\hat{\varepsilon}_{\text{RS}}(s, \lambda) &= 4.51 \times 10^{-39} Z^2 \left( \frac{k_B T_e}{E_{\text{Ryd}}} \right)^{1/2} n_Z n_e \bar{S}_{\text{RS}} \ .
\end{align*}
\]

(13)

where \(\alpha\) indicates the beam energy components. \(\bar{S}\) are the Stokes vectors for the different light emission processes. Except for MSE, the light is assumed to be unpolarized. The \(\mathcal{P}\) are normalized profile functions reflecting the shapes of spectral lines as observable from the plasma. \(\mathcal{P}\) is a function of the coordinate \(s\) along the line of sight, \(\lambda\), and—in case of discrete lines—of the line centers. \(Z\) is the effective charge of the plasma, \(E_{\text{Ryd}}\) is the Rydberg energy of hydrogen. The Einstein coefficients \(A\) refer to upper levels as indicated.

At this point, a discussion of the components of the emission coefficients is helpful. The MSE component is the most prominent part of a performance study. The charge-exchange (CX or FIDA) neutrals result in broad line contributions potentially overlapping with the MSE multiplet. Therefore, it is crucial to consider the charge-exchange part of the spectrum. Neutral particles moving into the observation volume forming a beam halo certainly affect the spectrum but are considered to be on the order of magnitude of charge-exchange contributions or even less. The detailed description of the halo contribution is subject to later model extensions. Here, it will be modeled by an additional broad charge-exchange component. The emission of cold edge neutrals is large but can be suppressed in the spectrometer. To predict diagnostic performance, complicated mechanisms like the fast ion \(D_a\) emission require comparable high modeling effort but are known from experiments to result in a small contribution. Therefore, they will be left out in this study. Impurity lines are assumed to be separated and do not affect the spectrum in the MSE region. The bremsstrahlung background \(\varepsilon_{\text{RS}}\) is small in amplitude, but the small broadband contribution summing up over the entire spectrum affects the global uncertainty of the forward model considerably.

Now, the remaining part for the simulation of MSE spectra in future devices is the signal amplitude. According to Eqs. (7) through (11), the signal amplitude requires the calculation of the densities of light-emitting particles. This can be done by a CRM as depicted in Fig. 2 showing the levels and their respective population and depopulation paths. Main excitation processes are heavy particle collisions along with electron collisions. Moreover, the proton collisions also lead to the population of neutrals by charge exchange and excitation transfer. Beam attenuation is due to beam ionization and, again, charge exchange. Radiative processes lead to the decay of excited particles and are used to quantify the spectroscopic signal. The resulting CRM contains all densities \(\tilde{n}\) of considered states both in the beam and the plasma:

\[
\frac{d}{dx} \tilde{n} = \frac{1}{v_B} A \tilde{n} \ ,
\]

(14)

\(H^0\) beam
\(H^0\) plasma

Fig. 2. Simplified Grotrian diagram representing the CRM for the simulation of MSE spectra.
where $v_b$ is the beam velocity and $A$ contains the rate coefficients for the collisional and radiative processes as indicated in Fig. 2 to be solved for the beam components $\alpha$ at full, half, and third energies. The transition rates depend on the plasma parameters.

Solving the CRM is necessary to predict signal heights and their dynamics along the beam. For this simulation, the stationary solution ($\partial f / \partial t = 0$) of Eq. (14) is employed. For data analysis, it is feasible to describe the signal amplitude by a fit parameter. Once additional data on densities and temperatures and information on the plasma equilibrium became available, a more comprehensive fit became possible as demonstrated in the concept of integrated data analysis.\(^5\)

II.B. . . . To Forward Calculation

Now, a signal can be calculated from equilibrium information $\bar{B}(\vec{x})$ and profiles of $n_e(r), n_i(r), T_e(r), T_i(r), Z_f(r)$, and $E_r(r) = (\nabla \phi(r))$. The equilibrium yields the mapping from flux surfaces to real space coordinates $\vec{x} = \vec{x}(r)$ and metric coefficients $|g|$ to calculate real space radial electric fields $\tilde{E}_r(\vec{x}) = (\partial \phi(r) / \partial r)\tilde{\vec{r}}$.

To determine beam-related effects, the beam velocity $\tilde{v}_b$, its divergence angle $\gamma_b$, and the density composition in full beam energy $E_b, \text{half beam energy } E/2$, and third beam energy $E/3$ is taken as an experimental parameter as well as the port-through power $P_b$.

The CRM [Eq. (14)] is solved as an initial value problem with all beam particles being in the ground state before the beam enters the plasma. The rate coefficients as functions of densities and temperatures were taken from fit expressions.\(^2\) The solution gives the densities $n_\text{beam}^{(n=3, \pi)}(s), n_\text{beam}^{(n=3, \sigma^+)}(s)$ for Eq. (7), and $n_\text{plasma}(s)$ for Eq. (8) assuming a statistical multiplet distribution. More sophisticated CRMs, e.g., to account for alignment effects,\(^18,19\) could be used instead.

To complete the necessary information for Eq. (7), the Einstein coefficients for the sublevels $\Lambda_\alpha$ and $\Lambda_{\sigma^\pm}$ are taken from solutions of the Schrödinger equation.\(^28\) The profile $P_b$ is the Doppler width of the multiplet lines resulting from the beam divergence from a Gaussian intensity profile of the injected beam. This broadening has been found in experiments to be the leading broadening effect, but more detailed assumptions can be added straight-forward. The line shifts of the multiplet components $\lambda_\alpha$ and $\lambda_{\sigma^\pm}$ result from the line splitting due to the electric field

$$ \vec{F}(s(\vec{x})) = \tilde{v}_b \times \vec{B}(\vec{x}) + \tilde{E}_r(\vec{x}) $$

and a Doppler shift due to the beam motion. The line splitting is calculated from the Schwarzschild-Epstein formula. The Doppler shift is determined from the line-of-sight and the beam geometries.

For the charge-exchange component, the line emission profile $P_p$ is determined from the ion temperature; the Doppler shift due to rotation is not taken into account for W7-X simulations but is allowed to occur for the tokamak data. The Einstein coefficient $A_{\pi=3}$ is summed from the multiplet-resolved Einstein coefficients.

The polarization state is determined from the sums in Eq. (7); i.e., the $\pi$ components are linearly polarized with respect to the quantization axis given by $\vec{F}$. Accordingly, the $\sigma^\pm$ components are circularly polarized in the direction of $\vec{F}$. The charge-exchange emission and the bremsstrahlung are assumed to be unpolarized.

At this point, the polarization state–resolved and wavelength-dependent local emission coefficient $\varepsilon(s, \lambda)$ can be determined. The line-of-sight integration $\int_{s} \Delta \varepsilon[s, \text{pixel} (\lambda)] \, ds$ is considered by the length of the optical path transversing the beam. With given etendue $T$ and pixel sensitivity $\sigma$, the polarization-dependent transmission $\Pi$ is left to determine the data $D$ in Eq. (4).

Since the forward model will result in partially polarized light, $\Pi$ is formulated as a chain of Müller matrices applied to the Stokes vector $\vec{S}$. The definition follows Ref. 24:

$$ \vec{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} (E_0 E_0^* + E_1 E_1^*) \\ (E_0 E_1^* - E_1 E_0^*) \\ (E_0 E_1^* + E_1 E_0^*) \\ i(E_0 E_1^* - E_1 E_0^*) \end{bmatrix} $$

where $I$ represents the total radiance and $Q, U$, and $V$ are quantities describing the linearly and circularly polarized contributions of the total radiance. The electric field $\vec{E}$ of the observed light is given by

$$ \vec{E} = \vec{E}_0 \exp(i(k_z - \omega t)) $$

and

$$ \vec{E}_0 = E_0 \hat{e}_x + E_1 \hat{e}_z $$

where $\parallel$ and $\perp$ are orthogonal axes, both of which have components perpendicular to the light propagation.

In a frame given by a quantization axis $\hat{e}_z$ parallel to the electric field $\vec{F}$, the electric fields of light from $\Delta m = 0 (\pi)$ and $\Delta m = \pm 1 (\sigma)$ lines are

$$ \vec{E}_\pi = E_z \hat{e}_z = 2^{-1/2} E_0 \exp(-i \omega t) \hat{e}_z $$

and

$$ \vec{E}_\sigma = E_x \hat{e}_x + E_z \hat{e}_z $$

$$ = \frac{1}{2} E_0 \exp(-i \omega t) \hat{e}_z \pm i \exp(-i \omega t) \hat{e}_x $$.

The resulting Stokes vectors are

$$ \vec{S}_\sigma = \frac{1}{2} I_0 \begin{bmatrix} \cos^2 \theta \\ \cos^2 \theta \\ 0 \\ 0 \end{bmatrix} $$

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where \( \theta \) is the angle between the line of sight and \( \tilde{F} \). \( S_{\pi} \) reflects the emission anisotropy of the emission from a Hertzian dipole. For the \( \sigma^{\pm} \) components one arrives at

\[
S_{\sigma^{\pm}} = \frac{1}{4} I_0 \begin{pmatrix}
1 + \sin^2 \theta \\
1 + \sin^2 \theta \\
0 \\
\pm 2 \sin \theta
\end{pmatrix}.
\]  

(21)

It is interesting to note that the Stokes vector \( \vec{S}_z \) given by

\[
\vec{S}_z = \vec{S}_\pi + \vec{S}_{\sigma^+} + \vec{S}_{\sigma^-}
\]

reflects Heisenberg’s principle of optical stability: If the radiances of degenerate \( \pi \) and \( \sigma \) lines are emitted such that

\[
\sum \sigma_i A_{\sigma_i} = \sum \sigma_{i\sigma^+} A_{\sigma_{i\sigma^+}} = \sum \sigma_{i\sigma^-} A_{\sigma_{i\sigma^-}}
\]

then the emission is isotropic; i.e., \( \vec{S}_z \) is independent from the observation direction expressed by \( \theta \). Although all elementary emission processes result in polarized light, the total emission is unpolarized for \( Q, U, V = 0 \) in Eq. (22). Wavelength separation due to external fields, however, changes the situation fundamentally. It is also interesting to note that the formulation of the Stokes vector contains any effect of emission anisotropy, which is sometimes also reflected by the sublevel resolved Einstein coefficients in textbooks. However, deviations from statistical and II.


III. FORWARD SIMULATION OF MSE SPECTRA FOR W7-X

Now, the forward model will be applied to cases for W7-X. First, the density of light-emitting particles is calculated. The required input quantities (refer to Table II) are taken from predictive transport simulations and settings. Geometrical data were taken from computer-aided design (CAD) data. A CAD view of a possible MSE arrangement is shown in Fig. 3. Table I summarizes derived quantities in the model and the measurand; Table II contains the input parameters of the model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Data</td>
<td>From calibration</td>
</tr>
<tr>
<td>pixel</td>
<td>Argument of data</td>
<td>Measurand</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Wavelength</td>
<td>Derived quantity</td>
</tr>
<tr>
<td>( \tilde{F} )</td>
<td>Electric field</td>
<td>Derived quantity</td>
</tr>
<tr>
<td>( \tilde{\theta} )</td>
<td>Angle of ( \tilde{F} ) with line of sight</td>
<td>Derived quantity</td>
</tr>
<tr>
<td>n_{source}</td>
<td>CRM state vector</td>
<td>Derived quantity</td>
</tr>
<tr>
<td>( \vec{S} )</td>
<td>Stokes vector density of state by emission source</td>
<td>Derived quantity</td>
</tr>
<tr>
<td>( I, Q, U, V )</td>
<td>Stokes vector components</td>
<td>Derived quantity</td>
</tr>
<tr>
<td>( L )</td>
<td>Radiance</td>
<td>Derived quantity</td>
</tr>
<tr>
<td>( e )</td>
<td>Emission coefficient</td>
<td>Derived quantity</td>
</tr>
</tbody>
</table>
With this input, the simplified CRM has been solved. The CRM used here was truncated to \( n = 3 \) to estimate the population densities in the light-emitting states (MSE) and to simulate the beam attenuation (charge exchange). For analyses, this model underestimates the \( n = 3 \) population and cannot resolve nonstatistical distributions in sublevels. More detailed CRMs for beam particle populations are reported, e.g., in Refs. 19 and 33. For design purposes, however, the model yields reasonable rates of light emission and the required dynamic detector resolution if the attenuated beam is observed along the beam line. A beam-stopping model for deuterium beams has been reported in Ref. 34. The simple model used here, however, is computationally fast allowing one to apply feasible optimization algorithms. Some results for the state-resolved CRM in total poloidal and toroidal plasma and beam states are shown in Fig. 4. For the simulation of a 60-keV hydrogen beam, >90% of the beam is deposited in the plasma. The main attenuation processes are charge-exchange collisions and ionizing collisions with

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**TABLE II**

Summary of Input Model Parameters (Settings) for the MSE Forward Model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_e, n_i )</td>
<td>Electron/ion density</td>
<td>Transport model</td>
</tr>
<tr>
<td>( T_e, T_i )</td>
<td>Electron/ion temperature</td>
<td>Transport model</td>
</tr>
<tr>
<td>( Z )</td>
<td>Effective ion charge</td>
<td>Transport model</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>Electrostatic potential</td>
<td>Transport model</td>
</tr>
<tr>
<td>( E_r )</td>
<td>Radial electric field</td>
<td>Transport model</td>
</tr>
<tr>
<td>( B )</td>
<td>Magnetic field vector</td>
<td>Transport model</td>
</tr>
<tr>
<td>(</td>
<td>g</td>
<td>)</td>
</tr>
<tr>
<td>( e_r )</td>
<td>Flux surface normal vector</td>
<td>Equilibrium</td>
</tr>
<tr>
<td>( r = r(\tilde{x}) )</td>
<td>Mapping</td>
<td>Equilibrium</td>
</tr>
<tr>
<td>( n, n_1, n_2 )</td>
<td>Quantum numbers</td>
<td>Multipllet</td>
</tr>
<tr>
<td>( A_{ji} )</td>
<td>Rate coefficients for CRM</td>
<td>Atomic data, transport model</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>CRM transition matrix</td>
<td>Atomic data, transport model</td>
</tr>
<tr>
<td>( A_{\sigma} )</td>
<td>Einstein coefficients</td>
<td>Atomic data</td>
</tr>
<tr>
<td>( \tilde{x} )</td>
<td>Real space coordinate for observation volume</td>
<td>Setting (beam line, line of sight)</td>
</tr>
<tr>
<td>( s )</td>
<td>Beam coordinate, line-of-sight coordinate</td>
<td>Setting (beam line, line of sight)</td>
</tr>
<tr>
<td>( P_b )</td>
<td>Beam power</td>
<td>Setting (NBI)</td>
</tr>
<tr>
<td>( \tilde{E} )</td>
<td>Beam energy</td>
<td>Setting (NBI)</td>
</tr>
<tr>
<td>( \tilde{v}_b, v_b )</td>
<td>Beam velocity ( \tilde{E} )</td>
<td>Setting (NBI)</td>
</tr>
<tr>
<td>( \alpha_{E,E'}/\alpha_{E'}/E/3 )</td>
<td>Beam composition</td>
<td>Setting (NBI)</td>
</tr>
<tr>
<td>( \gamma_b )</td>
<td>Beam divergence</td>
<td>Setting (NBI)</td>
</tr>
<tr>
<td>( M )</td>
<td>Müller matrix</td>
<td>Setting (observation)</td>
</tr>
<tr>
<td>( \Pi )</td>
<td>Chain of Müller matrices</td>
<td>Setting (observation)</td>
</tr>
<tr>
<td>( s )</td>
<td>CCD sensitivity</td>
<td>Setting (observation)</td>
</tr>
<tr>
<td>( A_s )</td>
<td>Spectrometer entrance area</td>
<td>Setting (observation)</td>
</tr>
<tr>
<td>( \Omega_s )</td>
<td>Minimum solid angle of detector</td>
<td>Setting (observation)</td>
</tr>
<tr>
<td>( T )</td>
<td>Etendue</td>
<td>Setting (observation)</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Observation angle</td>
<td>Setting (observation)</td>
</tr>
<tr>
<td>( \mathcal{P} )</td>
<td>Line profile</td>
<td>Setting, plasma</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Transmission</td>
<td>Setting (observation)</td>
</tr>
</tbody>
</table>

---

Fig. 3. Three-dimensional CAD view on the diagnostic beam line and possible observation geometry of a spectral MSE measurement on W7-X (as used for the simulations).
heavy particles. Yielding the measurand, the $n = 3$ densities are shown indicating a sharp rise in the emission intensity when the beam enters the plasma, decreasing to about one order of magnitude in the course of the beam. The more spurious light emission from $n = 3$ plasma atoms (due to charge-exchange processes) shifts to the right along the beam coordinate. Once the $n = 3$ plasma emission has overcome the $n = 3$ beam emission, the $n = 3$ plasma emission multiplet remains larger than the corresponding beam emission.

For a cross-check of the figures shown in Fig. 4, the maximum fraction of $n = 3$ beam particles (at $T = 2.8$ keV and $n \approx 8 \times 10^{19}$ m$^{-3}$) was found to be $n_3/n_1 = 0.29\%$. Comparable calculations for stationary situations\(^{35}\) give fractions of $n_3/n_1 \approx 0.28\%$ for $T = 2.8$ keV and $n \approx 10^{19}$ m$^{-3}$. In the plasma region, the simple model gives $\sim 15\%$ larger values than Ref. 35, indicating the error because of neglecting levels $n > 3$. For the design of diagnostics, the results indicate quantitatively the required dynamic range for light detection.

Motional Stark effect spectra for the geometry of the diagnostic beam planned for W7-X are shown in Figs. 5 and 6. The plots contain spectra for 16 spatial channels corresponding to the beam penetrating from lower beam coordinates; the beam emission of the outermost two channels is not excited due to vanishing plasma densities for the respective positions. For a representative discussion, we consider a channel corresponding to the gradient region of the plasma (bold line in Figs. 5 and 6). Finite beam effects enter the simulations through the beam divergence only. Further refinement of the simulation with regard to finite beam and finite observation cones are expected to broaden the multiplet emission further. A more detailed neutral beam model is required, e.g., for ITER, where the resulting emission profile of the individual multiplet components should even be expected to get structured rather than being a single-mode (Gaussian) profile.\(^{36}\)

Figures 5a and 5b show the effect of the emission anisotropy and the observation geometry. The spectra consist of dominant peaks (for the discussed channel on the blue-shifted side) and a broad charge-exchange contribution. The dominant peaks correspond to the central $\sigma$ lines of the full-, half-, and third-energy components of the beam. Being broadened because of the beam divergence, the $\sigma$ and $\pi$ components cannot be resolved as separated lines. While the total number of emission processes is shown in Fig. 5a, the MSE lines significantly decrease in Fig. 5b reflecting the different emission characteristics of the differently polarized components. The unpolarized charge-exchange contribution, however, is unaffected (except for the etendue). The apparent ratio of dominant peaks, moreover, is changing. This indicates the contribution of $\pi$ lines from the full energy to overlap with the $\sigma$ lines of the half-energy spectrum. Thus, the overlapping components of different polarizations from different beam energies may lead to misinterpretation if only the radiance at the maximum of the lines is considered.

Figures 5b and 5c show the effect of a reduced (half) beam divergence. Now, the $\pi$ components become visible, but again, the apparent relative ratios of the dominant peaks change, indicating again the necessity to account for all multiplet components properly.

To study the effect of polarizing elements, Fig. 6 shows the spectra after transversing different polarizers. Similar to the previously discussed cases, the effect of polarizing elements affects the line ratios. It is concluded from these results that MSE diagnostics require a quantitative consideration of both the polarization of emission as well as the polarizing properties of the observation optics.
Finally, Fig. 7 shows the effect of nonstatistical distributions. Density-dependent corrections for the upper state distributions have been taken from results of an elaborate CRM \cite{Ref. 19} at $T/\text{H}^{1000} = 3 \text{ keV}$, $E/\text{H}^{1000} = 50 \text{ keV}$, and $F/\text{beam}$. The parameters of the density differ somewhat from the parameters in our simulations \cite{Ref. 101} at $T/\text{H}^{4000} = 6 \text{ keV}$, $E/\text{H}^{6000} = 60 \text{ keV}$, but the difference in the line ratios is considered to be representative. It can be seen that the radiance maximum of the central $\sigma$ component is $\sim 10\%$ smaller while the $\pi$ components become up to $15\%$ larger because of nonstatistical multiplets. The effect must be expected to increase if the densities become smaller than here ($n < 8 \times 10^{19} \text{ m}^{-3}$). As a conclusion, the effect of nonstatistical distributions is a correction on the order of $10\%$ for the simulation of signals. For analyses, as shown in Sec. IV, the ratio of the $\pi$ and $\sigma$ components is a sensitive fit parameter. Consequently, a correct interpretation of this fit parameter needs to consider nonstatistical distributions properly.

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Fig. 5. Simulation of MSE spectra for W7-X (60-keV $H^{10}$ diagnostics beam, beam intensity ratios $I_E/I_{E/2}:I_{E/3} = 0.7:0.15:0.15$, beam divergence 1 deg). The panels show different observation channels; the total shift corresponds to the beam coordinate reflecting the Doppler shift of the spectrum. (a) shows the total emission from the plasma, (b) shows the effect of emission anisotropy and rotation, and (c) corresponds to (b) but at half of the beam divergence. The labels in (c) indicate, respectively, the peaks mainly from $\sigma$ lines of full-, half-, and third-energy beam emission. The unpolarized charge-exchange component is indicated by CX. The peak positions apply for this figure and Fig. 6.

Fig. 6. Simulation of MSE spectra observation (corresponds to Fig. 5b); $\nu$ and $h$ are vertical and horizontal polarizers, respectively.

Finally, Fig. 7 shows the effect of nonstatistical distributions. Density-dependent corrections for the upper state distributions have been taken from results of an elaborate CRM (Ref. 19) at $T = 3 \text{ keV}$, $E = 50 \text{ keV}$, $F \perp \text{beam}$. The parameters of the density differ somewhat from the parameters in our simulations ($T/\text{H} \approx 4 \text{ keV}$, $E = 60 \text{ keV}$), but the difference in the line ratios is considered to be representative. It can be seen that the radiance maximum of the central $\sigma$ component is $\sim 10\%$ smaller while the $\pi$ components become up to $15\%$ larger because of nonstatistical multiplets. The effect must be expected to increase if the densities become smaller than here ($n < 8 \times 10^{19} \text{ m}^{-3}$). As a conclusion, the effect of nonstatistical distributions is a correction on the order of $10\%$ for the simulation of signals. For analyses, as shown in Sec. IV, the ratio of the $\pi$ and $\sigma$ components is a sensitive fit parameter. Consequently, a correct interpretation of this fit parameter needs to consider nonstatistical distributions properly.
IV. VALIDATION OF THE FORWARD MODEL ON ASDEX UPGRADE

To check the validity of the forward model, Eq. (5) has been fitted to experimental spectra. The setup of the measurement is described in Ref. 26. The purpose of the fit is to reproduce the shape of the spectra and to determine the value of the electric (Lorentz) field. Therefore, and since an absolute calibration of the observation is underway, the CRM has not been solved for the fit, but the beam intensity is taken into account by a fit factor.

Figure 8 shows results for MSE spectra from ASDEX Upgrade. Gross features of a typical MSE spectrum can be clearly identified. The apparent difference in the number of peaks compared to the W7-X simulation (refer to Fig. 6) is due to different viewing geometries and beam energies making some π components appear as additional distinct lines. The MSE lines lie in the spectral region of ∼653 to 655.5 nm. To suppress the high radiance \( \text{D}_a \) radiation from the plasma edge, a wire has been placed in the focal plane of the spectrometer as indicated by the shaded areas in Fig. 8. Nevertheless, charge-exchange processes contribute to the spectrum with a broad emission reflecting the ion temperature. The fit gives a central ion temperature of ∼1.2 keV for the observation channels in the plasma center. To attain a good fit to the data, a second contribution at lower temperature needs to be included. Both contributions are allowed to be Doppler shifted reflecting plasma rotation. A more detailed assessment of

---

**Fig. 7.** Simulation of MSE spectra with nonstatistical multiplets (to be compared with Fig. 5c). (b) shows the difference of spectra from statistical (Fig. 5c) to nonstatistical distribution (a).

**Fig. 8.** Experimental MSE spectra and fits of the forward model to the data. (b) shows the residuum.
the broad background lines is beyond the scope of this paper. As a third part of the spectrum, background bremsstrahlung is subtracted as a constant offset. Carbon impurity lines have been fitted; these lines seem to indicate plasma rotation as well.

Summarizing, the fit consists of three main parameters: the electric field, a polarization factor, and the beam composition. The polarization factor is sought to reflect the direction of the electric field. It has been introduced to vary the relative intensities of $\pi$ and $\sigma$ components entirely for each polarization state. This single factor was found to be sufficient for the fit. The value of the resulting factor, however, cannot be explained by emission anisotropy only. Deviations might have their origin in atomic physics effects and/or the effect of polarizers in the observation optics (polarization-dependent transmission). In order to disentangle the different effects, calibration measurements are underway.

The resulting electric field from the spectra can be compared to the Lorentz field from equilibrium calculations [CLISTE (Ref. 37)]. The MSE electric field is $\sim 10\%$ systematically larger than the results from unconstrained equilibrium calculations. Such discrepancy has been reported in Ref. 38 and is presently under assessment. As the residuum (Fig. 8b) indicates, the present forward model shows systematic deviations in the blue side wing of the MSE component. This may indicate the influence of fast ion $D_e$ emission. Furthermore, the residuum shows oscillatory deviations in the region of the Stark multiplet. This is sought to result from non-Gaussian beam profiles of the neutral beam. From fits, a mean total effective divergence of $1.57 \pm 0.12$ deg ($1/e$) has been derived. From the specified minimum divergence of the ASDEX Upgrade neutral beam injection (NBI) (1.05 deg) (Ref. 40), the beam focusing geometry (0.40 $\pm 0.06$ deg), and a rough estimate of the acceptance angle of the observation optics observed (0.70 $\pm 0.26$ deg), one arrives at a harmonic sum of 1.34 $\pm 0.16$ deg, with an error also considering the apparatus width of the detection system. The values agree within a 1$\sigma$ error estimate. A more detailed neutral beam modeling with regard to the profile shape is under investigation. A NeI line at $\sim 650.5$ nm from wavelength calibration can be found in all spectra (not fitted).

Concluding, the fit of the forward model gave very reasonable agreement with the data (normalized $\chi^2/N \approx 10$; the error of the data has been determined by calibration measurements at varying radiance), indicating the leading-order effects to be compliant with the choice of contributions in the W7-X forward model. Deviations have been figured out and are addressed for diagnostic improvements.

V. SUMMARY

A forward model for MSE spectra has been implemented. Simulations for spectra for W7-X have been performed. The required dynamic range of a detector can be derived from results of the CRM. The influence of observation geometry and polarizing effects in the detection optics shows the necessity to include the polarization properties both of emission and detection for the W7-X example. Diagnostic amendments, e.g., improvements of beam divergence, exemplify possible studies for diagnostic designs. Meeting with the requirement to enhance the confidence in the design, the validity of the model has been assessed by fitting the forward model to experimental data from ASDEX Upgrade. The model resulted in reasonable fits; this confirms the choice of leading-order effects in the simulation.

The physics structure of the forward model implies a modular structure of the synthetic diagnostic software. Main parts of model modules are the beam-plasma interaction, physics of local light emission, and spectroscopic detection. The model structure defines interface requirements for the subparts of the model. The implementation of the MSE forward model as a prototype of validated synthetic diagnostics employing unified software interfaces and reusable software modules is underway.

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REFERENCES

5.6 Article VI

Motional Stark Effect Measurements of the Local Magnetic Field in High Temperature Fusion Plasmas
R. C. Wolf, A. Bock, O. Ford, R. Reimer, A. Burckhart, A. Dinklage, J. Hobirk, J. Howard, M. Reich, and J. Stober
Journal of Instrumentation 10, P10008 (2015)
A shorter version of this contribution is due to be published in PoS at:
1st EPS conference on Plasma Diagnostics

KEYWORDS: Plasma diagnostics - interferometry, spectroscopy and imaging; Nuclear instruments and methods for hot plasma diagnostics
1 Introduction

Understanding magnetic confinement of high temperature fusion plasma, i.e. plasma transport and stability, requires the detailed knowledge of the magnetic field inside the plasma. In stellarators, the magnetic field is provided, at least to a large extent, by external magnetic field coils. Nevertheless, at finite $\beta$ plasma currents modify the vacuum magnetic field and, as a consequence, also the confinement properties. In tokamaks, the poloidal magnetic field generated by the toroidal plasma current distribution forms an essential part of the confining magnetic field. Its intrinsic coupling to the plasma properties such as electrical conductivity or plasma transport makes measurements of the magnetic field distribution an essential tool for understanding tokamak plasmas.

Since the early days of fusion research, non-invasive diagnostic techniques to infer the magnetic field measurement inside plasma have been the focus of intensive developments (see e.g. review by Soltwisch [1]). The problem with all these techniques is, however, that the first order of the magnetic field distribution is generally not of interest. Typically, the equilibrium current distribution or safety factor profile in a tokamak can be derived from the knowledge of the pressure profile and magnetic probes measuring the field outside the plasma. In stellarators, the vacuum magnetic field, which already confines the plasma, can be even measured without plasma, employing electron beam techniques [2]. However, the interesting details of plasma transport or stability often depend on small deviations from these first order safety factor or rotational transform profiles. Also the diamagnetic reduction of the magnetic field is only of the order of a few percent. To measure these small variations, the corresponding technique must offer an appropriate sensitivity and accuracy.
The Motional Stark Effect (MSE) observed on high energy neutrals injected into the plasma turned out to be one of the most powerful methods to measure the magnetic field distribution inside high temperature plasmas. In the older literature, pointing out the diagnostic potential, the Motional Stark Effect is often referred to as translational Stark Effect [3, 4]. Prior to that, the effect was described in the context of fast hydrogen atoms entering the plasma through charge exchange with energetic protons [5]. First introduced on the tokamaks PBX-M [6] and JET [7] in 1989, today MSE diagnostics can be found on many fusion devices including tokamaks [8–10] and stellarators (or heliotrons) [11, 12]. Employing hydrogen or deuterium neutral beams (or in a few cases on JET also tritium beams), the linear Stark effect generated by the Lorentz electric field, \( \mathbf{E}_L = \mathbf{v} \times \mathbf{B} \), in the frame of the fast hydrogen moving with the beam velocity \( \mathbf{v} \) with respect to the background magnetic field \( \mathbf{B} \) produces a characteristic emission line pattern. MSE diagnostics normally use the Stark-split Balmer-\( \alpha \) transition \((n = 3 \rightarrow 2)\) which basically consists of nine emission lines with a characteristic wavelength splitting and \( \sigma \)- and \( \pi \)-polarization. The relative intensities of the remaining lines is so small (between 0.02 and 0.3%) that their contribution can be neglected for all practical purposes [13]. The polarization of the line emission and thus the ratio between \( \sigma \)- and \( \pi \)-polarized lines are sensitive to the orientation of \( \mathbf{E}_L \), while the wavelength splitting is determined by its strength. Translating this into a measurement of the magnetic field, the orientation of \( \mathbf{B} \) can be inferred from the measurement of the polarization or the \( \sigma \)- to \( \pi \)-ratio, while the wavelength splitting contains information about the magnetic field strength. Of course both, \( \mathbf{B} \) and \( |\mathbf{B}| \) depend on poloidal and toroidal magnetic field components, however in a different way. As a result, they show different sensitivities to changes of \( B_{\text{pol}} \) and \( B_{\text{tor}} \). Since the radial electric field in fusion plasmas, \( |\mathbf{E}_r| \), is typically two orders of magnitude smaller than \( |\mathbf{E}_L| \), it only has a significant influence on the MSE polarization measurement, provided the angle between \( \mathbf{E}_r \) and \( \mathbf{E}_L \) is large enough.

Many magnetic confinement fusion experiments which are also equipped with a neutral beam injection system employ a multi-channel MSE diagnostic to measure the radial profile of the magnetic field. Most such systems focus on the polarization measurement for the determination of the toroidal current density profile. In a few cases also spectrally resolved MSE diagnostics are used to infer \( |\mathbf{B}| \) and in some cases also \( \mathbf{B} \) from the \( \sigma \)- to \( \pi \)-line ratio. Despite the wide distribution of this technique, it still suffers from a number of issues related to the desired accuracy of such measurements. Calibrating MSE diagnostics and, in particular, polarization measurements in a fusion experiment environment is still challenging. In view of the increasing accuracy with which the spectral and polarization information can be measured, many inconsistencies or inaccuracies in the previous analysis become apparent. These include atomic physics effects such as the correct treatment of the population densities of the atomic levels, which determine the line intensities [14–16], or the admixture of the Zeeman Effect to the Stark Effect [4, 16]. There is also strong evidence that metallic plasma facing walls result in polarized reflections from the plasma background radiation leading to spurious polarization signals in the MSE diagnostic [17].

This paper is organized as follows: first the Motional Stark Effect and its basic features as plasma diagnostic are briefly introduced. Two basic measurement techniques are discussed. The first relies purely on the analysis of wavelength dependent emission distribution of the MSE spectrum, including the measurements of line splitting and line ratios. The second employs polarization measurements utilizing the characteristic polarization information contained in the MSE emission.
Finally, before concluding the paper, the so-called Imaging MSE diagnostic, which is a special form of polarization measurement [18], will be presented.

2 Motional Stark Effect

The Stark Effect describes the removal of the degeneracy of the atomic levels due to an external electric field superimposed to the atomic Coulomb field. The linear Stark effect only appears if the atomic levels are fully degenerate as it is the case for hydrogen. In the translational or Motional Stark Effect the electric field is caused by the $\mathbf{v} \times \mathbf{B}$ electric field (Lorentz electric field) appearing in the rest frame of an atom moving with the velocity $\mathbf{v}$ through a magnetic field $\mathbf{B}$. In fusion research powerful neutral heating beams, using hydrogen or its isotopes deuterium (and in a few cases at TFTR and JET also tritium [19, 20]), or dedicated diagnostic beams with high power densities have facilitated the development of MSE diagnostics. With beam energies of the order of 100 keV and magnetic fields of several Tesla, $|\mathbf{E}_L|$ lies in the range of $10^6$ to $10^7$ V/m. In the case of the Balmer-$\alpha$ $n = 3 \rightarrow 2$ transition, the $n = 3$ level splits into five and the $n = 2$ into three energy levels. Altogether, the selection rules produce nine distinct transitions which can be measured [7].

Figure 1. Typical setup of a beam emission diagnostic (here ASDEX Upgrade MSE setup). The beam or beams (magenta) are intersected by a fan of lines of sight (coloured from blue to red). The approximate orientation of the magnetic field (blue) and the corresponding Lorentz electric field (red) are also shown. At the position of the beam, a plasma equilibrium showing the flux surface contours and the separatrix is overlaid.

The emitted light originates from the interaction of the neutral beam with the plasma [21, 22]. The neutral atoms are excited by collisions with plasma particles and the radiative de-excitation generates the characteristic beam emission. As the neutral beam particles are also ionized by beam-plasma collisions, the beam is successively attenuated when penetrating the plasma. Higher
path-integrated electron densities corresponding to larger plasma cross-sections or injection more
tangential to the toroidal axis of the device result in stronger attenuation. As a consequence, the
intensity of the beam emission decreases limiting the applicability as a diagnostic. The other pa-
rameters determining the beam attenuation are the beam energies, which typically lie in the range
between 50 and 500 keV, and the energy composition, which in the case of positive ion beam
sources consists of three energy species at full, half and third energy.

Figure 2. Example MSE spectrum using deuterium beams measured at ASDEX Upgrade (pulse 26323 at
2.97 s, major radius $R = 1.86$ m, vertical position $Z = 0.09$ m). Measurement (Exp) and fit using a forward
model (Mod) are superimposed. Clearly visible are the three energy components $E_0$, $E_{1/2}$ and $E_{1/3}$. The
MSE multiplet is evaluated assuming a superposition of Stark and Zeeman Effects (ZMSE). Other spectral
features are: active and passive charge-exchange emission (CX), fast ion D-α component (FIDA) and CII
dge emission (Imp). In this measurement the Balmer-α edge emission has been optically blocked to avoid
over-exposure of the CCD detector.

Beam and viewing geometries influence the characteristics of the spectrum and the way the
measurement can be analysed. Typical viewing and beam geometries are shown in figure 1. To
spectrally separate the beam emission from other plasma emission, originating from the same atomic transition, the beams are observed at angles different from 90°. In the example shown, the lines of sight look into the beam resulting in a blue shift of the beam emission spectrum. An example MSE spectrum, measured at ASDEX Upgrade, is shown in figure 2. With nine MSE lines per energy component, altogether 27 blue-shifted lines are required to describe the spectrum. The spatial resolution of the diagnostic is given by the cross-section of the lines of sight and the volume defined by the lines of sight crossing the beam diameter. Depending on the dimensions of the beam with respect to the plasma dimensions and the variations of the plasma parameters of interest along the line of sight through the beam, the measurement can be regarded as local with some averaging of the plasma parameters along the emission volume. However, in particular in smaller fusion experiments, using relatively large high power heating beams, inversion techniques might become necessary to infer the plasma parameters from the line integrals through the beam diameter. Using a forward model to describe the measured data, the line integration can be implemented in a straightforward way [23]. In fact, the spectral fit in figure 2 uses such an approach. Finally, in some cases also the influence of the neutral beam on the plasma has to be considered. If the beam, which is used to heat and sustain the plasma, is also used for the MSE measurement, this is not an issue.
In the case where the beam is only applied for diagnostic purposes, dedicated diagnostic beams with lower power but high power density are required. If they are not available, some experiments use short beam blips to minimize the effect of the beam on the plasma [24].

3 Measurement of spectral properties

3.1 Measurement techniques

As an example, the observation system of the ASDEX Upgrade spectroscopic MSE diagnostic is described [25]. Figure 3 shows the observation geometry and the setup of the spectral and the polarimeter measurements. Similar systems can be found on many fusion experiments [12, 22, 26, 27]. Near the plasma boundary, a mirror is used to reflect the light collected from the neutral beam onto a more radial path. Subsequently, a lens system focuses the light onto a fibre bundle which relays the light to a spectrometer which generates the spectra for different radial locations. On ASDEX Upgrade, a window in front of the mirror protects the mirror against coating from plasma impurities and thus maintains its reflection properties. A dielectric mirror, designed for high reflectivity near the Balmer-α wavelength and for the reflection angles required, guarantees optimal reflection properties also for the differently polarized spectral lines. The beam emission spectra are recorded by a CCD camera. At ASDEX Upgrade, the full spectrum including the Balmer-α edge emission is recorded. Since this spectral emission is usually very strong, the centre of the line is blocked out by a thin metal wire which in the exit plane of the spectrometer is positioned exactly at the wavelength of this line. Keeping the wavelength range around the edge emission in the spectrum has the advantage that the charge exchange components of the spectrum reaching below the MSE spectrum and the Doppler shift between MSE energy components and edge emission are better constrained.

3.2 Spectral properties and analysis

Each energy component of the Stark multiplet consists of σ- and π-lines (see figure 2). The six π-lines are polarized parallel to the projection of \( E_L \), while the three σ-lines are an incoherent superposition of left and right hand elliptically polarized emission. Thus, observed parallel to \( E_L \) the σ-emission is unpolarized, observed perpendicular to \( E_L \) it becomes linear. Inbetween the σ-emission is linearly polarized with an unpolarized background. The ratio of the σ- to π-emission characteristic is given by \( (1 + \cos^2 \Theta) / \sin^2 \Theta \), where \( \Theta \) is the angle between \( E_L \) and the line of sight. In addition to this emission, characteristic line intensities depend on the population densities of the \( n = 3 \) levels and the transition probabilities. The latter can be easily derived from the atomic physics of the Stark effect. However, already early on it was discovered that the assumption of a statistical population of the upper levels does not agree with the observed line ratios [22]. More recent measurements show a clear density dependence of the relative intensities of the Stark multiplet [14]. Theoretical considerations can explain this by a non-statistical population of the atomic levels involved [15, 28]. Even for plasma densities of a few \( 10^{20} \) m\(^{-3} \), a statistical population distribution is not achieved. For calibration purposes, the beam is often injected into the plasma vessel filled with neutral gas. Also in this case, the populations of the \( n = 3 \) states is actually far from a statistical distribution [28].
The MSE setup at ASDEX Upgrade consists of an observation system, viewing one of the neutral heating beams, and dedicated elements for either spectral or polarization measurements. Up to ten radial channels along the beam axis are observed. For the measurement, the collected light is relayed by optical fibres to a Czerny Turner spectrometer which images the spectra onto a CCD detector. The polarization measurement uses photoelastic modulators (PEMs), a polarizer and photomultipliers. Interference filters select the spectral lines of interest. The details of the polarimeter setup are described in section 4.1. For testing the prototype Imaging MSE diagnostic, the last part of the optics including PEMs and polarizer are replaced by the imaging system.

The individual line widths of the Stark multiplet depend on the integration of the emission over the velocity distribution of the neutral beam particles along the line of sight \[29\]. This results in a dependence on beam and line of sight divergence which basically add up to an effective line width. Other effects which can contribute to the line broadening are variations of the magnetic field over the line of sight and the ripple of the acceleration voltage of the neutral beam injectors \[30, 31\].

The line splitting contains the information on the magnetic field strength. For the pure Stark effect, the line splitting is proportional to \(|E_L|\). In an idealized geometry (beam injection perpendicular to the magnetic field), this results in a direct proportionality to the total magnetic field strength \(|B| = (B_{\text{pol}}^2 + B_{\text{tor}}^2)^{1/2}\) (\(B_{\text{pol}}\) and \(B_{\text{tor}}\) are the poloidal and toroidal magnetic field components, respectively).

Although it was pointed out already in the work by Souw and Uhlenbusch \[4\] and later also in the PhD thesis by Yuh \[16\], the influence of the Zeemann Effect on the spectral properties of the MSE spectrum is usually neglected. Comparing the relevant terms in the Schrödinger equation \[22\], their ratio \((\mu_B B)/(3/2eE_La_0) = \mu_B/(3/2eva_0\sin \alpha)\) (where \(\mu_B\) is the Bohr magnetron, \(a_0\) the Bohr radius, \(v\) the beam velocity and \(\alpha\) the angle between \(B\) and \(v\)) is indeed considerably smaller than one. For ASDEX Upgrade, a rough estimate gives a ratio of about 0.3. However, the increasing accuracy of the method determining the spectral splitting requires a reconsideration of this issue. Since the Zeeman Effect also linearly depends on \(|B|\), it is not expected that measurements of
magnetic field changes are affected. However, for the determination of absolute values of $|B|$ the influence of the Zeeman Effect is significant.

### 3.3 Physical properties derived from MSE spectroscopy

The two basic measurements of MSE spectroscopy are that of the line splitting and of the ratio of $\sigma$- to $\pi$-components. In addition, the Doppler shift of the different energy components reveals information about the radial position of the observation volume and the line broadening can be used to infer the beam divergence [29, 30]. In fact, in [26] the temporal variation of the Doppler shift has been used to rule out any significant changes of the radial position of the observation volumes or of the beam velocity which could falsely hint magnetic field changes.

Assuming that the details of the beam geometry and beam velocity distribution are known, the line splitting can be used to derive the total magnetic field, $|B|$. At JET, this was implemented for the first time, inferring the diamagnetic reduction of the magnetic field due to ion cyclotron resonance heating [26]. Since the associated $\beta$-changes are of the order of a few percent, the required sensitivity must be in the per mill range. Knowing the pressure profile from kinetic measurements of plasma density and temperature, this technique can be used to infer the fast ion pressure contribution to the total pressure. Other early measurements also showed that — assuming the toroidal field is known — the inferred radial poloidal field distribution is in agreement with equilibrium calculations which use edge magnetic flux measurements as a constraint [22]. The data analysis originally employed a multi-Gaussian fit to the MSE spectrum using the coupling of parameters such as wavelength splitting and line ratios or their symmetries to minimize the free parameters as far as possible [22]. Recently, a forward model has been developed for the inference of the plasma parameters which also includes effects such as the fast ion contribution to the spectrum [25, 33].

The $\sigma$- to $\pi$-line ratios can be utilized to recover the orientation of the magnetic field, $B_{pol}/B_{tor}$, and from that quantities such as the current or safety factor profile [32]. Combining the $\sigma$- to $\pi$-ratio and the line splitting, allows to self-consistently derive direction ($B_{pol}/B_{tor}$) and magnitude ($|B|$) of the magnetic field [12, 27, 31]. Depending on the details of the measurement, special care has to be taken with the polarization dependent reflectivity of mirrors and the population density of the atomic levels. Both can significantly influence the outcome of the $\sigma$- to $\pi$-ratio line ratio measurement. Depending on the mirror properties and the alignment between $\sigma$- to $\pi$-polarization directions and p- and s-axes of the mirror, a large modification of the line ratio measurement can be produced which needs careful calibration [12]. A non-statistical population of the atomic levels introduces a density dependence of the line ratios which has to be included in the analysis of the spectral data [12, 33]. If the spectral resolution permits to resolve the Stark lines individually, the $\sigma \pm 1$ to $\pi \pm 3$ line ratios offer the possibility of a measurement independent of the relative population densities, as these transitions originate from the same upper level [22, 31]. Some diagnostic applications inject the beam into the plasma vessel filled with neutral gas to calibrate the line ratios in the presence of a known magnetic field without plasma and determine the influence of optical components, e.g. a possible mirror, on the measurement [12, 31]. In tokamaks, the vacuum field without plasma is the toroidal magnetic field which has a radial dependence $\sim 1/R$ (where $R$ is the major radius of the torus). In stellarators, the vacuum magnetic field already exhibits a rotational transform and can be calculated from the currents in the magnetic field coils. For beam into gas injection, the question about non-statistical population distribution is even more important [28].
While it has been reported that this calibration method in principle works [12], large discrepancies between predicted and measured $\sigma$- to $\pi$-ratios have been observed too [28].

**Figure 4.** Calculation of the magnetic field as a function of wavelength splitting, $\Delta \lambda$, for the ASDEX Upgrade MSE diagnostic (at radial a position corresponding to a major radius of $R = 1.90$ m) distinguishing two cases: MSE assumes pure Stark effect neglecting any Zeeman influence (solid lines). ZMSE is a solution of the Schrödinger including both Zeeman and Stark terms, but neglecting the spin-orbit coupling (dashed-dotted lines). The crosses stand for the actually calculated points corresponding to a magnetic field ramp performed during ASDEX Upgrade pulse 26322. The lines represent fits to these points. The three groups of lines correspond to the full (black), half (blue) and third energy components (red) of the beam. For given values of the wavelength splitting of all three energy components the magnetic field values are indicated by the horizontal lines for, both, MSE and ZMSE cases. Since in the ZMSE case the wavelength splitting also varies within each energy component, $\Delta \lambda$ represents the mean value of all Stark lines from $-4\pi$ to $+4\pi$.

Basically, following the calculations of Souw and Uhlenbusch [4], the combination of Motional Stark and Zeeman Effects has been revisited. Solving the Schrödinger equation in the strong magnetic field limit and neglecting the spin-orbit coupling, but including a Stark term, the modification of the wavelength splitting has been calculated. The coordinates have been chosen in such a way that according to $E_L = v \times B$ the electric field is perpendicular to the magnetic field. Generally, the wavelength splitting increases with respect to the pure Stark case. In figure 4, the calculated dependence of the magnetic field on the wavelength splitting is shown for an ASDEX Upgrade example. The apparent value of $|B|$, assuming Stark Effect only (MSE case in figure 4), is larger than the value inferred from the combination of Zeeman and Stark Effects (ZMSE case in figure 4). Depending on beam energy, the discrepancies for ASDEX Upgrade beam parameters and a magnetic field of 2.3 T are between 3% (for 20 keV deuterium; third energy component) and 1% (for 60 keV deuterium; full energy component). While the change of the wavelength splitting with magnetic field does not show a significant difference between the two cases, the deviations of the absolute magnetic field values between 1% and 3% are significant. As long as one looks only
at changes of the magnetic field, e.g. for the evaluation of the pressure change of the plasma, neglecting the Zeeman Effect does not affect the result. Both lines (MSE and ZMSE) are sufficiently parallel. This is not surprising as both Zeeman and Stark Effects linearly depend on $|\mathbf{B}|$. However, for an absolute measurement of $|\mathbf{B}|$ the misinterpretation without considering the Zeeman Effect is of the order of the plasma para- or diamagnetism.

4 Measurement of the polarized line emission

4.1 Measurement techniques

As outlined above, the spectral lines of the MSE multiplet are polarized. Rather than utilizing the different emission characteristics of the $\sigma$- and $\pi$-lines, MSE polarimetry tries to measure their polarization directions directly. The polarization measurements have the principle advantage that they do not depend measurably on the population distribution of the atomic levels. Up to now, four measurement techniques have been investigated.

The simplest technique combines two static beam splitting polarizers dividing the incoming light of each line of sight into two polarization components. For each polarization component the full MSE spectrum is recorded [22]. A multi-Gaussian fit is used to separate $\sigma$- and $\pi$-lines and subtract the unpolarized background signal. A half-wave plate in front of the light collecting optics makes sure that the incoming linearly polarization is approximately split into equal parts. As a result, the ratio of the $\pi$-emission from the two polarization components is a direct measure of the orientation of $\mathbf{E}_L$ and subsequently $B_{\text{pol}}/B_{\text{tor}}$ [34]. This technique has the advantage that the polarization can be measured while the full spectral information is retained. However, the requirement to record two Stark spectra for each line of sight with sufficient spectral resolution, including a meaningful background measurement, at the time of the measurement limited the possible time resolution. Nowadays, this problem could be overcome by using faster CDD detectors. In addition, the quality of the spectral analysis necessary to separate $\pi$- from $\sigma$-emission and the unpolarized background determines the sensitivity of the diagnostic to $\mathbf{E}_L$ variations as it is the case also for the measurement of the $\sigma$- and $\pi$-line ratios.

A technique proposed by Voslamber [35] overcomes the last problem. Instead of only using two static linear polarizers, this technique uses three linear polarizers and one circular polarizer for each line of sight which are aligned in such a way that the measured signal corresponds to the four Stokes parameters. Without the necessity of a sophisticated spectral analysis, these four parameters not only contain the information about the polarization of the MSE emission, but also can be used to recover the influence of polarizing elements in the transmission optics, such as a non-ideal mirror, without any extra calibration. To be sufficiently sensitive, the measurement only requires a spectral resolution which separates $\pi$- from $\sigma$-emission. However, additional effort is generated by the necessity to record up to four spectra per line of sight. An implementation of this technique in a slightly modified form has been installed and tested on the Large Helical Device (LHD) [11] measuring small variations of the rotational transform induced by neutral beam current drive. In this case, four linear polarizers were used at the angles covering an angular range from 0° to 135° in steps of 45°.

The today most commonly used technique was first introduced on the tokamak PBX-M [6]. The polarimeter consists of two so-called photo-elastic modulators (PEMs) [36]. A PEM gen-
erates a time dependent phase modulation by oscillating stress induced birefringence. The MSE polarimeter setup consists of two PEMs at different orientations with respect to their optical axes of birefringence and operating at slightly different frequencies. Typical frequencies of the PEMs used for the MSE polarimeters are 20 and 23 kHz. Adjusting the PEM amplitudes in such a way that the maximum phase shift corresponds $\lambda/2$, they work in a similar way as rotating half-wave plates. Followed by a linear polarizer, the whole assembly transforms any incoming linear polarization into an amplitude modulated signal with the following time response (for the general setup see figure 3): $I \propto \sin(2\gamma) \cos(2\omega_2 t) - \cos(2\gamma) \cos(2\omega_1 t) + \ldots$, where $\gamma$ is the incoming polarization angle and $\omega_1$ and $\omega_2$ are the two frequencies of the PEMs [6]. Taking the ratio of the two $\cos(\omega_i t)$ terms, $\tan(\gamma)$ can be recovered. The intensities at the second harmonic frequency of the PEMs can be measured either by lock-in amplifiers driven by the PEM frequencies or recording the full time evolution of the modulated signal. Subsequently, the modulation intensities at $2\omega_i t$ are inferred by Fourier transforming the measured signal.

In contrast to the first two techniques using static polarizers, the PEM-based dynamic polarimeter relies on the spectral filters to select either the $\sigma$- or the $\pi$-components of the spectrum. Usually, the central $\sigma$-component is selected as its central wavelength position only depends on the angle between line of sight and neutral beam and the beam velocity, which determine the Doppler shift, but not on the magnetic field strength. For beam energies between 50–100 keV and magnetic fields between 2–3 T, the width of the spectral filters must be in the range of 2–3 Å which requires temperature control to keep the filter at the desired wavelength. Only in spherical tokamaks with their smaller magnetic fields narrower filters have to be used [37, 38]. Since the Doppler shift causes the central wavelength of each Stark multiplet to differ for different lines of sight, each line of sight requires a different filter. PEM-based multi-channel MSE polarimeters have been installed on many tokamaks [37–45].

Crucial for any MSE polarimeter is the absolute accuracy of the polarization angle measurement and the calibration to achieve this [46]. Issues involved with the calibration of the polarimeter setups are the Faraday rotation induced by the magnetic field inside the observation optics, reflections from (non-ideal) mirrors, any type of spurious birefringence in the observation optics, or coatings of first mirrors or windows from plasma vessel conditioning or from plasma operation. If the orientation of the optical axis of the observation optics is strictly radial with respect to the magnetic field, Faraday rotation does not occur. However, the optics are usually between the toroidal field coils where the corresponding field is radial and very inhomogeneous. Further out, the radial field from the poloidal field coils can also be significant. For most optical setups, the optical axis exhibits at least a small component in the direction of the magnetic field. Special glass with a low Verdet constant can be used to minimize the effect [47]. In doing so, typical values of the Faraday rotation of the ASDEX Upgrade MSE polarimeter are of the order of $1^\circ/T$. In fact, the remaining $1^\circ/T$ is from the protection window in front of the mirror which is still made of fused silica. In the case of DIII-D, even lower values have been achieved [48]. If the Faraday rotation remains an issue for the measurement, the effect can be minimized by inserting a half-wave plate in the path of the collection optics. Considering incident light with arbitrary polarization,

$$\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} |E_x|e^{i\delta_x} \\ |E_y|e^{i\delta_y} \end{pmatrix},$$
where $E_x$ and $E_y$ are the components of the electric field vector in a Cartesian coordinate system and correspondingly $\delta_x$ and $\delta_y$ are arbitrary phase shifts, the effect of the Faraday rotation can be described as

$$\vec{E}' = \begin{pmatrix} \cos \alpha - \sin \alpha \\ \sin \alpha \cos \alpha \end{pmatrix} \vec{E} = \begin{pmatrix} E_x \cos \alpha - E_y \sin \alpha \\ E_x \sin \alpha + E_y \cos \alpha \end{pmatrix}.$$ 

The angle $\alpha$ represents the Faraday rotation in the first part of the optics. Introducing a half-wave plate, the electric field vector is transformed into

$$\vec{E}'' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \vec{E}' = \begin{pmatrix} E_x \cos \alpha - E_y \sin \alpha \\ -E_x \sin \alpha - E_y \cos \alpha \end{pmatrix}.$$ 

The coordinate axes $(x, y)$ represent the optical axes of the half-wave plate. If the electric field vector is rotated by another Faraday angle $\alpha + \Delta \alpha$ in a subsequent part of the optics, the light vector becomes

$$\vec{E}''' = \begin{pmatrix} \cos(\alpha + \Delta \alpha) & -\sin(\alpha + \Delta \alpha) \\ \sin(\alpha + \Delta \alpha) & \cos(\alpha + \Delta \alpha) \end{pmatrix} \vec{E}'' = \begin{pmatrix} E_x \cos(\Delta \alpha) + E_y \sin(\Delta \alpha) \\ E_x \sin(\Delta \alpha) - E_y \cos(\Delta \alpha) \end{pmatrix}.$$ 

Assuming that $\Delta \alpha$ is small the final light vector can be expressed as

$$\vec{E}''' = \begin{pmatrix} E_x \\ E_y \end{pmatrix} + \Delta \alpha \begin{pmatrix} E_x \\ -E_y \end{pmatrix}.$$ 

If the half wave plate is placed in such a way that the Faraday rotation angles in the first and second part of the optics are the same ($\Delta \alpha = 0$), the influence of the Faraday rotation essentially cancels. The sign of the $y$-component of the electric field has changed. However, this is same if the half-wave plate were placed in the optical path without Faraday rotation. To ensure that the method also works for a certain range of opening angles, a zero-order half-wave plate has to be used. The minimization of the Faraday rotation not only eases the requirement for a possible calibration of the effect with respect to changes of the magnetic field of the fusion experiment, but also avoids possible problems with the temperature dependence of the Verdet constant. An ideal mirror (with high reflectivity) in principle has the same effect as the half-wave plate. Many MSE diagnostics use such mirrors to image the beam onto the light collecting optics. However, the positions of those mirrors do not necessarily fulfill the condition $\Delta \alpha = 0$.

In order to minimize the influence of the mirror on the polarization measurement, in ASDEX Upgrade the mirror is provided with a dielectric coating which for the wavelength range and the angular range given maximizes the reflectivity [49]. In addition, in ASDEX Upgrade the mirror is protected against coatings by an exchangeable glass window in front of the mirror. In fact, thin deposits were found on this window, which could serve as a possible explanation for observed drifts of the offset calibration of the polarization angles.

Comparing the independently measured polarization angles of the $\sigma$- and $\pi$-components by tuning the wavelength filters to the corresponding wavelengths, a dependence of the difference of the $\sigma$- and $\pi$-polarization angles on the plasma density has been found on ASDEX Upgrade. Without any modification of the polarization angles or other spurious polarization contributions, this difference should be $90^\circ$. However, above line integrated densities of about $6 \times 10^{19}$ m$^{-3}$
Figure 5. Dependence of the difference of the $\sigma$- and $\pi$-polarization angles on the line integrated plasma density, measured through the plasma centre of ASDEX Upgrade. Data points are from the innermost and outermost radial channels of the MSE polarimeter of more than a hundred discharges looking only at the neutral beam for which the diagnostic was designed. In theory, the difference of the measured polarization angles from the $\sigma$- and $\pi$-components, $\Delta \gamma_m = \gamma_\sigma - \gamma_\pi$, should be $90^\circ$. The deviations by up to $15^\circ$ at higher densities could be an indication for broadband polarized background radiation contaminating the measurement generated by plasma radiation reflected from the metal walls. However, there are also examples which show little or no deviations (ASDEX Upgrade pulse 31163) despite large density variations.

deviations from this value of up to $15^\circ$ are observed. The effect is illustrated in figure 5. While the reason is still unclear, this behaviour seems to coincide with the introduction of tungsten as a plasma facing material. The question is whether polarized plasma radiation lies within the wavelength band of the polarimeter or whether the wall reflections turn unpolarized background radiation, e.g. bremsstrahlung, into partially polarized light [17]. Measurements of the wall-reflections inside Alcator C-Mod with an unpolarized light source indeed show a large polarization fraction favouring the second explanation. Also beam modulation experiments with plasma show a polarization signal during the beam-off times [17].

4.2 Physical properties derived from MSE polarimetry

In tokamaks, the main objective of the polarimeter systems is to measure the poloidal magnetic field distribution or the current density profile which are crucial quantities determining confinement and stability. In stellarators (or heliotrons), the quantities of interest are currents driven by heating systems or bootstrap currents. To test physical models, both, the measurements in tokamaks and stellarators require a very high absolute accuracy typically corresponding to magnetic field pitch
angle \( (\arctan \frac{B_{\text{pol}}}{B_{\text{tor}}}) \) errors between 0.5° and 0.1°. Considering all possible error sources, the lower value maybe at the limit what is technically achievable.

A famous example is the sawtooth instability occurring in tokamaks and related to this the formation and possible reconnection of internal magnetic islands. Although regularly observed, a conclusive theoretical model does not exist until today. Only recently, theoretical advances give an explanation for the fast reconnection time observed in tokamaks [50]. Closely related to the reconnection process is the behaviour of the safety factor profile. Crucial for the understanding is the temporal evolution of the safety factor at the centre of the plasma, \( q_0 \), and whether it drops significantly below 1 or rises above 1 just after the sawtooth crash. However, up to now the experimental evidence is inconsistent supporting both, full reconnection (\( q_0 \) rises above 1) [51] or partial reconnection (\( q_0 \) stays below 1) [52, 53].

Another important physical quantity which can be derived from an MSE polarimetry measurement is the radial electric field, \( E_r \) [43, 54, 55]. In tokamaks and stellarators, the radial electric field plays a central role in the plasma transport and in the understanding of confinement transitions. Despite the small value of \( |E_r| \), \( E_r \) can make significant contribution to the measured orientation of \( E_L + E_r \), in particular because for the usual MSE diagnostic setups \( E_r \) is approximately orthogonal to \( E_L \). One possible way to extract \( E_r \) from the polarization measurement is to utilize the first and the second energy component simultaneously thus introducing another independent measurement of the polarization angle with different beam velocity values. Another possibility is to employ two differently oriented beams at the same beam energy, using the fact that now the observation angles between line of sight and the neutral beams are different.

Nowadays, many tokamak applications use MSE data as a constraint for equilibrium codes. Thereby, the measured polarization angles serve as a strong constraint for the current density profile [56]. Using this approach, also other quantities can be derived. For instance, the non-inductive plasma current profile can be determined by computing the temporal evolution of the poloidal flux profile [57] derived from such equilibrium calculations.

## 5 Imaging MSE

Imaging MSE refers to a new technique [18] which up to now has been tested on TEXTOR [58], KSTAR [45] and ASDEX Upgrade [59–61]. The system works like a static polarimeter capable of producing a full 2-dimensional image of the polarized plasma emission [62]. It consists of a conventional imaging system with a CCD or CMOS type detector. A wavelength filter is chosen in such a way that a large fraction of the MSE spectrum (\( \sigma \)- and \( \pi \)-components and also parts of second or third energy component) is transmitted, while the edge Balmer-\( \alpha \) is blocked out. Birefringent plates, introduced in the optical path, generate a wavelength and polarization dependent phase shift transforming the incoming polarization information into an interference pattern:

\[
I \propto 1 + \zeta \cos(2\gamma) \cos(\omega_x) + \zeta \sin(2\gamma) \cos(\omega_x + \omega_y) + \zeta \sin(2\gamma) \cos(\omega_x - \omega_y),
\]

where \( I \) is the measured intensity as function of the image coordinates \( x \) and \( y \), \( \omega_x \) and \( \omega_y \) are approximately constant, and \( \zeta \) is the so-called spectral contrast, which is a slowly varying function depending on the optical properties of the birefringent plates and spectral distribution of the incoming light. Fourier-transforming the image, \( I \), and separating the components yield the polarization
Figure 6. (6a) shows the beam emission image, superimposed with the interference pattern. The Fourier transform, (6b), exhibits the three components which contain the information about the polarization angle. (6c) Plotting the polarization angle as a function of $x$ and $y$ yields a 2-dimensional polarization distribution.

Figure 7. (7a): in-vessel background image recorded with the ASDEX Upgrade prototype Imaging MSE diagnostic showing in-vessel structures and plasma facing components. In (7b) the image of the transformed polarization angle, $\theta$, in $(R,Z)$-coordinates at the beam intersection plane is superimposed with the flux surface contours. The polarization image reaches from the plasma edge (normalized flux $\psi_N = 1$) to the centre ($\psi_N = 0$).

angle $\gamma$ (projection of the orientation of $E_L$ onto a plane perpendicular the viewing direction) as a function of the image coordinates. An example of an ASDEX Upgrade measurement [60] is illustrated in figure 6.

A clear advantage of the Imaging MSE technique is the wealth of information gained by recording emission images and at the same time having a 2-dimensional measurement of the polarization angles. Since in contrast to the PEM-based polarimeter the bigger part of the spectrum is used, the signal to noise ratio is very good. Typical filters cover all spectral lines of the full energy components and parts of the second and even third energy component and, therefore, have a width which is about an order of magnitude broader than that required for the PEM-based MSE polarimetry. The imaging technique allows observing different beams or more than one beam simultaneously. In addition, background images of e.g. the plasma vessel walls can be recorded. Together with the beam emission image, this defines the viewing geometry. Figure 7a) shows the background image recorded with the ASDEX Upgrade Imaging MSE prototype diagnostic [60]. In-vessel structures and plasma facing components are clearly visible. Points and dashed lines indicate reference points for the spatial orientation. The diagonal structure in the lower right part of
Figure 8. Temporal evolution of the polarization angle as measured (8a) by the PEM-based MSE system and (8b) by the Imaging MSE diagnostic in near-identical pulses with similar sawtooth activity. The temporal resolution of the PEM data is 8 ms, that of the Imaging MSE data 5 ms. For the analysis of the Imaging MSE data radial positions and image areas, over which the data have been averaged, have been chosen in such way that they correspond to the positions and spatial resolution of the PEM-based polarimeter. In (8c) the radial derivative of the polarization angle near the plasma centre is shown, confirming that the sawtooth collapse involves a fast redistribution of plasma current.

The image is a poloidal limiter. In figure 7b) the transformed polarization image in \((R,Z)\) at the beam intersection plane is superimposed with the flux surface contours also indicating the normalized flux label of the surfaces.

However, to get away from plasma dependent calibrations and achieving the desired accuracy requires an understanding of all polarimeter details, such as the behaviour of the spectral contrast which is assumed to exactly cancel when taking the ratio of the Fourier components. Issues which remain also for an Imaging MSE diagnostic are the Faraday rotation calibration or the possible effects of mirrors or spurious birefringence from the optical elements such as vacuum windows. Another advantage of an Imaging MSE system is that it is not affected by a spectrally broadband polarized emission which is suspected to influence PEM-based systems (as shown in figure 5). Polarization effects from narrow band plasma emission in the passband would be apparent as a disturbance to the expected Doppler phase shift and could be masked during the signal processing.

With the ASDEX Upgrade prototype Imaging MSE system, good agreement of the measurements with both, theoretical predictions and the PEM-based MSE polarimeter has been achieved [61]. The time resolution and improved signal to noise allows the resolution of sawtooth oscillations, which has not been possible with the PEM-based MSE system. Figure 8a) and b) show the evolution of the polarization angle as measured by the two systems for near-identical pulses in which similar sawtooth activity is confirmed by soft X-Ray measurements. It is clear that the signal to noise ratio of the PEM-based MSE is insufficient to observe the sawtooth signature. Figure 8c) shows that the radial derivative of the polarization angle near the plasma centre also
exhibits the sawtooth evolution in the Imaging MSE data. This quantity is approximately related to the local plasma current and gives a direct confirmation that the sawtooth collapse involves a fast redistribution of plasma current. However, the determination of the evolution of the central q-value can only be achieved with an exceptionally good absolute calibration, which is the focus of ongoing work.

6 Summary and conclusions

The Motional-Stark-Effect is the most widely used technique to gain information about the internal magnetic field distribution and dynamics of magnetic fusion experiments. Measurements of the full MSE spectrum can be used to derive both the magnetic field strength and its orientation. Aiming at absolute accuracies of $|B|$ of the order of 0.1%, the influence of the Zeeman Effect has to be considered as the corresponding corrections are of the order of 1%. $B/|B|$ can be inferred from a line ratio measurement which either relies on the knowledge of the details of the population density distribution or a very good spectral resolution to accurately resolve the individual line intensities of the Stark multiplet. Polarimeters make use of the polarization information contained in the Stark emission to derive $B/|B|$. The most commonly used technique is a dynamic PEM-based polarimeter which combines high time resolution with a comparatively simple data analysis. Imaging MSE systems have been introduced recently showing significant advantages. However, the full understanding of these systems is still progressing. After successfully testing a prototype on ASDEX Upgrade, a dedicated Imaging MSE diagnostic has been developed and installed. First results are expected this year. However, all polarimeter techniques need elaborate calibration techniques to achieve the necessary accuracy. This difficulty is reflected e.g. in the fact that the question of partial or complete reconnection during the sawtooth cycle is still unresolved, although progress has been made with resolving sawtooth oscillations with the Imaging MSE prototype.

A frequently asked question is how to implement an MSE diagnostic on ITER. Studies propose both a spectroscopic system [63] and a polarimeter [64]. As was investigated earlier [22], the first study suggests using the measurement of the line splitting to derive the tokamak q-profile thus avoiding detrimental effects of the large number of mirrors required to protect the vacuum window and the subsequent optical elements against neutron radiation. It would be certainly interesting to investigate whether an Imaging MSE diagnostic would solve some of the issues related to a polarimetric measurement on ITER.

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5. Thesis Articles


Selbständigkeitserklärung


(René Reimer)  Greifswald, den 10. August 2016
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